

Task Trade between Similar Countries*

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Abstract

We propose a theory of task trade between countries that have similar relative factor endowments but may differ in size. Firms produce differentiated goods by performing a continuum of tasks, each of which generates local spillovers. Tasks can be performed at home or abroad, but offshoring entails costs that vary by task. In equilibrium, the tasks with the highest offshoring costs may not be traded. Among the remainder, those with the relatively higher offshoring costs are performed in the country that has the higher wage and higher aggregate output. We discuss the relationship between equilibrium wages, equilibrium outputs, and relative country size and examine how the pattern of specialization reflects the key parameters of the model.

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1 Introduction

Modern production assigns a prominent role to international task trade. The delivery of a good or service to a consumer typically requires the completion of a myriad of different tasks. Increasingly, the performance of these tasks is spread across the globe, with an impressive share of offshore production in the value of many final goods. As a result, international trade is less today a matter of countries' specialization in particular industries and more about their specialization in particular occupations and tasks.

Much has been written about the growth of offshoring between countries that stand at different levels of development, i.e., countries that have dissimilar factor endowments and disparate technological capabilities.¹ Yet, as important as this sort of offshoring is becoming in world trade, it pales in comparison to task trade between similar countries. Not only does most trade flow between and among the advanced industrialized economies, but these economies are engaging in an ever more intricate web of production-sharing arrangements.

The Boeing 787 Dreamliner is a case in point.² The production of this new midsize jet involves 43 suppliers spread over 135 sites around the world. Boeing relies heavily on local expertise when making its sourcing decisions. The wings are produced in Japan, the engines in the United Kingdom and the United States, the flaps and ailerons in Canada and Australia, the fuselage in Japan, Italy and the United States, the horizontal stabilizers in Italy, the landing gear in France, and the doors in Sweden and France. Offshore production accounts for close to 70 per cent of the many thousands of parts used to assemble the jet (Newhouse, 2007, p.29). Some parts are produced in foreign affiliates of the Boeing Corporation while others are supplied under international outsourcing agreements. The countries that perform the various tasks display no clear pattern of technological advantage. Rather, experience and local knowledge play a central role. Apparently, expertise most often derives from similar tasks being performed for other Boeing projects or for related industries, such as military aviation and automobile production.³

Aggregate data on production sharing among the developed countries is difficult to come by.⁴ Yet, hints of the substantial magnitude of such task trade abound. As one example, we point to the location of the stocks of U.S. foreign direct investment and of the employment of foreign affiliates of U.S. firms. Figure 1 shows the U.S. direct investment position in several regions of the world. Not only is Europe the site of more than half of the accumulated foreign assets of U.S. firms, but the most recent FDI flows are adding to its lead. Canada, another "similar" country in terms of

¹See, for example, Jones and Kierzkowski (1990, 2001), Feenstra and Hanson (1996), Deardorff (2001a, 2001b), Yi (2004), Egger and Falkinger (2003), Kohler (2004a, 2004b), Baldwin and Robert-Nicoud (2007), and Rodriguez-Clare (2007), as well as our own work in Grossman and Rossi-Hansberg (2008b).

²The sourcing of Boeing's parts for the 787 is detailed at <http://www.boeing.com/commercial/787family/background.html>. See Newhouse (2007) for further discussion.

³See "A Cleverer Way to Build a Boeing," *The Financial Times*, 07/08/2007.

⁴Many researchers have documented production sharing for particular countries in their trade with the rest of the world; see, for example, Campa and Goldberg (1997), Hummels, Rapoport and Yi (1998), Yeats (2001), Hummels, Ishii and Yi (2001), Amiti and Wei (2005, 2006), and Hanson, Mataloni and Slaughter (2005). However, none of these authors identifies the share of task trade that takes place between countries at similar stages of development.

relative factor endowments and technological capabilities, accounts for an additional ten percent of the accumulated foreign investment. All told, more than 60 percent of U.S. FDI resides in Europe and Canada. Figure 2 shows the geographic spread of employment in foreign affiliates of U.S. firms since 1997, the first year for which such data are available. Clearly, employment in Europe dwarfs that in the other locations. Not all of the activity in foreign affiliates of U.S. firms represents offshoring, nor does production sharing necessarily require an ownership relationship. Still, the figures on FDI and employment in foreign affiliates suggest the importance of similar countries as partners in U.S. task trade.

Figure 1: U.S. Direct Investment Position Abroad on a Historical-Cost Basis

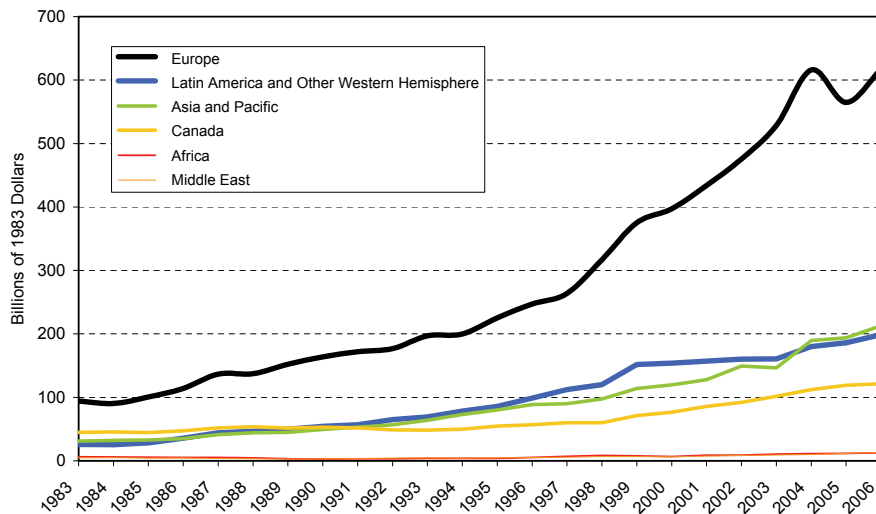
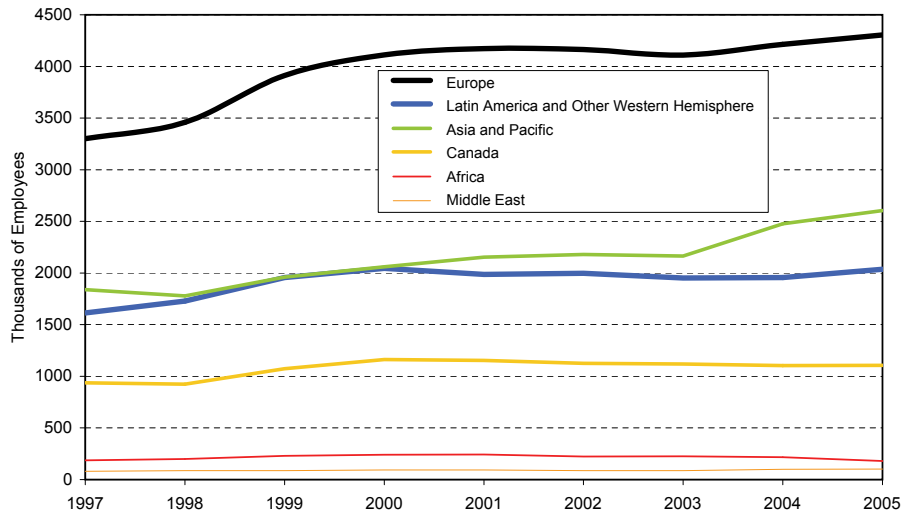


Figure 2: Employment of all U.S. Foreign Affiliates



In this paper, we formulate a theory of task trade between similar countries. In our model, firms incur entry costs to develop the know-how to produce particular goods. Production of any good requires the completion of a continuum of tasks. The set of required tasks is the same for all goods, yet the resulting products are differentiated in the eyes of consumers. Producers of the final goods engage in monopolistic competition and sell their wares to consumers who hold constant-elasticity-of-substitution (CES) preferences.

In keeping with the anecdotal evidence cited above, our treatment of production sharing emphasizes the role of local knowledge and specialized expertise. Our approach shares with the “new trade theory” a focus on increasing returns to scale as a force that induces concentration of production. But whereas the most familiar models in that literature feature trade in final goods—for which scale economies internal to the firm may be most pertinent—our focus on task trade dictates a different approach. The expertise to produce a unique good may well reside in a single firm, but the expertise to perform a narrow task rarely does so. Rather, it is often embodied in a pool of specialized labor, be they engineers with specific training or workers with shared experience. This suggests that localized knowledge at the task level may reflect external economies of scale rather than (or in addition to) internal economies. We take this notion to the extreme by assuming that productivity in performing a task varies with the frequency with which it is performed in a particular location, irrespective of the identify of the firm or firms performing the function.

The location of each task balances two competing forces. On the one hand, the external economies of scale provide firms with an incentive to locate each task in the country where others are performing it. On the other hand, it is costly for firms to organize and monitor the performance of tasks in countries different from where their headquarters are located. Our model features heterogeneous offshoring costs to capture the reality that some tasks are easier to separate from firms’ headquarters than others. For example, routine tasks can be performed remotely at relatively little extra cost, because instructions can be expressed unambiguously and conveyed easily to workers, with little need for interaction with central management. Other tasks may require greater adaptation to circumstances, so proximity to headquarters may be more important.⁵ Our analysis links the pattern of specialization by task to the distribution of offshoring costs.

When small firms operate in an environment with external economies of scale, they face an obvious coordination problem. If other firms are performing an activity in some location, it may be most profitable to join them there, even if all other economic forces point to a different outcome. As a result of the potential coordination failures, multiple equilibria can arise. Multiplicity of equilibrium has plagued models with production externalities, where “history” and “expectations” play a role in determining final outcomes. With a continuum of tasks and the possibility for self-fulfilling expectations for each of them, it might appear that little could be said about the equilibrium allocation of tasks across countries. Yet the environment with many tasks suggests a solution to the coordination problem that narrows the set of equilibria dramatically. In particular,

⁵Autor et al. (2003) have emphasized this distinction between routine and non-routine tasks and provided a measure of this concept. Levy and Murnane (2004) and Grossman and Rossi-Hansberg (2008) have applied the concept to explain variation in the costs of offshoring.

we recognize that firms can perform tasks on behalf of others. The opportunity for a firm to perform a task for many producers means that it potentially can internalize the externalities of locational choice. Such a supplier need not be large in relation to its industry, because even if it dominates the performance of a particular task, there are many other tasks to be done. By introducing the possibility for outsourcing tasks, we find that we can cut through the coordination issues and say quite a lot about the pattern of specialization in equilibrium.

Our main proposition relates the pattern of specialization by task to equilibrium relative wages and equilibrium aggregate outputs. It states that all firms perform the tasks that are most costly to offshore in the country of their headquarters. Among the remaining tasks, those that are easiest to offshore concentrate in the country that has lower wages and lesser aggregate output, while those that are more difficult to offshore concentrate in the country that has higher wages and greater aggregate output. Depending on the overall level of offshoring costs, the general equilibrium may be unique or not. With offshoring costs sufficiently high that identical-sized countries would engage in no production sharing, the unique equilibrium with unequal-sized countries has higher wages in the larger country. When offshoring costs are not so high, there will be multiple equilibria in a world with countries of nearly identical size, but a unique equilibrium when country sizes diverge. In the former case, there is one equilibrium in which wages are higher in the (slightly) larger country, another in which wages are higher in the (slightly) smaller country, and a third equilibrium with equal wages. In the latter case, the (much) larger country enjoys the higher wages.

Our finding is best understood by thinking first about efficiency considerations. The costs of communication and coordination can outweigh the potential gains from agglomeration for tasks that are quite costly to offshore. These tasks are efficiently performed in both locations. The other tasks should be concentrated in one location, which means they will generate offshoring costs for one set of producers. Aggregate costs can be minimized if the tasks that are most costly to offshore are performed in the country with the larger number of producers or the greater output per firm, because firms headquartered in this country perform these tasks the greatest total number of times. Since not all tasks can be performed in one country, it is efficient to locate those with modest offshoring costs in the country with fewer firms or less output per firm. We will find that the market allocation that results from potential internalization by large providers of tasks is not always efficient, because deviant suppliers can appeal to one set of national producers without taking into account the harm they cause to others. Still, the competitive forces mimic the socially optimal ones, at least qualitatively. Moreover, the equilibrium allocation of the difficult-to-offshore tasks to the country with greater aggregate output gives this country a cost advantage in producing final goods. This, in turn, justifies its greater scale of output per firm and also a higher equilibrium wage.

Once we have proven our basic characterization of the equilibrium allocation, we rely on numerical computations to explore the relationship between equilibrium outcomes and the key parameters of the model. In particular, we study the connection between the pattern of specialization and the extent of increasing returns to scale, the extent of product differentiation, and the size of offshoring

costs. We find that larger size differences between countries generate a broader range of traded tasks and imply larger wage differentials as long as some tasks are performed in both countries. Stronger external economies of scale and higher elasticities of substitution have similar implications for the extent of production sharing and for relative wages. Not surprisingly, a reduction in offshoring costs induces more task trade and tends to improve welfare.

We are not aware of other efforts to explain the pattern of task trade between similar countries in the nascent literature on offshoring. There have, of course, been many attempts in recent years to understand the high volume of goods trade between countries at similar levels of development. The early writings on product variety and trade (e.g., Dixit and Norman, 1980, Krugman, 1979, and Helpman, 1981) were designed exactly for this purpose, but had little to say about the pattern of specialization and trade. Trade patterns between similar countries have been the focus of research on external economies of scale at the industry level, on trade in differentiated products bearing transport costs, and on comparative advantage that derives from differences in the distribution of factor endowments for countries that share similar aggregate endowment ratios. This last approach (e.g., Grossman and Maggi, 2000, and Ohnsorge and Treffer, 2007) is quite different from the one pursued here, so we do not discuss it any further.

Markusen and Melvin (1981) and Ethier (1982) were the first to explore the determinants of the trade pattern in general-equilibrium settings with Marshallian production externalities.⁶ Both considered two-sector economies with constant returns to scale in one industry and increasing returns to scale due to external economies in the other. Both established the existence of a stable equilibrium in which the larger country specializes in and exports the good produced with increasing returns. Although their results are superficially similar to ours, the underlying economics are quite different. In their models, stability (and efficiency) dictate concentration of the increasing-returns industry in a single country. The smaller country may lack sufficient resources to satisfy world demand for this good, even if it is completely specialized, whereas the larger country always can do so.⁷ In our context of task trade, the ability to accommodate world demand never is at issue, because any small task can easily be concentrated in either location. The pattern of specialization does not rest on country size *per se*, but rather on the interplay between the scale of aggregate production of final goods and the offshoring costs.

Trade costs feature prominently in the literature on “home-market effects,” which began with Krugman (1980) and Helpman and Krugman (1985). The latter studied a world economy producing differentiated varieties subject to internal increasing returns to scale and a homogenous good with constant returns to scale. When the differentiated products are costly to trade and the homogeneous product is not, the large country exports the former and imports the latter. The larger country has a larger home market, and the shipping costs translate proximity to consumers into a profitability

⁶See, also, Melvin (1969) for an early contribution, Krugman (1991) and Matsuyama (1991) on dynamic stability issues, and Helpman (1984) for a survey and further discussion.

⁷In Grossman and Rossi-Hansberg (2008a), we study a trade model with Marshallian externalities in a continuum of final-goods industries. In such a setting, country size plays no role in determining the chain of comparative advantage. It does, however, effect the cut-off between goods that are produced in each country.

advantage for local firms. Davis (1995) points out that the home-market result rests on the assumed asymmetry in transport costs across sectors; if the homogenous good is as costly to trade as the differentiated products, sectoral trade between similar economies is balanced. Amity (1998) revisits the issue in a model with two increasing-returns industries that differ in trade costs. She shows that the larger country exports the goods that are more costly to ship, because its larger market provides local firms with a relatively greater advantage in that industry. Holmes and Stevens (2004) and Hanson and Xiang (2004) extend her result to a world with many industries and heterogeneous transport costs.

The results in the literature on market-size effects and heterogeneous trade costs resemble ours. In both cases, locational advantages give rise to factor-price differentials, as the country that bears the higher transport cost must offset this disadvantage in order that its factors be fully employed. In the literature on the home-market effect, the scale economies are internal to the firm and the cost differences stem solely from market size. In contrast, external economies of scale seem more pertinent for modeling production sharing. The recognition of such externalities requires us to address coordination issues and the role of producers who potentially can internalize these benefits. In our context, the scale economies drive each traded task to be performed in a single location, whereas in the models of internal economies of scale, incomplete specialization is the more typical outcome. Also, for task trade, the scale of final-goods production and not the location of final demand determines the pattern of specialization. Production of final goods is related to country size, but the country that produces more final output need not be the one that is larger in size.

The remainder of this paper is organized as follows. In Section 2, we develop our model of offshoring, discuss the equilibrium allocation of tasks given factor prices and aggregate outputs, and lay out the conditions for a general equilibrium. Section 3 begins with some illustrative examples that facilitate our consideration of the uniqueness of equilibrium and some equilibrium properties. The section proceeds to a discussion of the relationship between country size, aggregate output of final goods, and relative wages, and presents our main result on the pattern of specialization. In Section 4, we use numerical methods to study the relationship between the pattern of task trade and the key parameters of the model. Section 5 concludes.

2 The Model

Production requires many “tasks.” Each such task can be performed close to a firm’s national headquarters or at a foreign location. If a task is performed offshore, the firm bears an extra cost of coordinating production and communicating with distant workers. The cost of offshoring varies by task. Some require more frequent and intense interaction between workers and managers, while others are easier to perform from a distance.

We study an environment with external economies of scale at the task level. A firm’s productivity in performing a task in a particular location increases with the total scale of performance of the task by all firms in that same location. As in the literature on increasing returns to scale at

the industry level, the external economies are meant to capture the presence of localized knowledge spillovers.⁸

There are two countries, East and West. Each country is endowed with fixed supplies of two primary factors, managers and workers. In East, the supplies of these factors are H and L , respectively; in West, they are H^* and L^* . The similarity of the two countries is reflected in their identical relative factor supplies; $H/L = H^*/L^*$. However, the sizes of the two countries need not be the same.

A producer must perform (or procure) a unit measure of tasks to generate a unit of final output. The set of tasks required of different producers is the same, but their resulting outputs are differentiated in the eyes of consumers. Let $\sigma > 1$ be the elasticity of substitution between any pair of final products. The world market for these goods is characterized by monopolistic competition, with (constant) mark-up pricing and zero profits. We abstract from any cost of transporting final goods in order to highlight the costliness of offshoring.

The tasks that comprise a firm's variable cost are performed by workers alone. A firm can perform a task locally or offshore, and it can do so either in-house or by outsourcing the task to another firm. In addition to the production (or procurement) costs, each firm must hire managers to oversee production and coordinate the performance of the various tasks. A firm requires f managers in the country of its headquarters as a fixed cost of doing business. By paying this fixed cost, it gains the capacity to perform the continuum of tasks in a set of locations of its choice.

In this paper, we do not address the choice between vertical integration and outsourcing. Instead, we assume that firms use the same technology when performing tasks for themselves as when performing them for others. Moreover, firms must pay a small extra cost to acquire the capability to serve as an external provider of a task. In equilibrium, no firm has any incentive to pay this cost, so all tasks are performed in-house. Notwithstanding this outcome, the potential for outsourcing plays a meaningful role in our analysis. A firm that can perform a task for many others can (partially) internalize the externality associated with the choice of location. Although the equilibria that we describe feature no outsourcing, the possibility that a single firm might perform a task for others eliminates many potential equilibria and allows us to characterize the allocation of tasks across national boundaries.

We model the siting of each task as a multi-stage game. In the first stage post entry, firms choose—for each task i —whether to pay the cost that would prepare them to serve as a supplier. The capability to outsource a given task requires a small number ϕ of additional managers per unit of task. A firm that bears this cost for task i can perform the task on behalf of any or all other producers. A firm that does not incur the cost cannot supply the task to others. At the same time, each firm selects a “tentative” location for task i . This location can be changed at the next stage, but only at a (small) cost. In the second stage, firms that have the capacity to serve as suppliers quote prices. Since the tasks are performed specifically for a final producer, we allow

⁸On this point, see for example the discussions in Marshall (1920), Helpman (1984), Romer (1986), and Lucas (2002), among others.

for price discrimination; that is, a supplier may quote one price to perform the task on behalf of firms headquartered in the East and a different price to perform the task for firms headquartered in the West. The prices include any offshoring costs (which are described further below). At the same time that prices are quoted, firms choose their “final” locations for each task. If a firm’s final location differs from the tentative location chosen at the prior stage, then its variable cost is a multiple λ (slightly in excess of one) times as great as what it otherwise would have been. In the final stage of the siting game, each firm decides whether to perform task i for itself or to procure the task from the supplier that has offered it the lowest price.

There are external economies in the performance of every task that impart increasing returns to scale at the national level. Suppose that task i is performed a total of X_{ij} times in some country j . Then a firm that has its headquarters in country j and that chose country j as its tentative and final location for task i can perform the task in-house with $1/A(X_{ij})$ workers per unit of output, where $A(\cdot)$ is continuously differentiable, increasing, and concave. The labor requirement is the same for a firm that has invested in outsourcing capability and that seeks to perform task i in country j for other firms headquartered there. If, instead, a firm performs task i in country j for a producer (itself or another) with headquarters in country j' , and if country j was also its tentative location for task i , then it bears the (higher) per unit labor requirement $\beta t(i)/A(X_{ij})$. Here, $\beta t(i) > 1$ reflects the cost of offshoring task i . All of the labor requirements are multiple by λ for firms that switch their location between the initial and final designation. These switching costs afford potential suppliers the opportunity to make profits in case the firms tentatively coordinate on the “wrong” location.

Our modeling of offshoring costs mirrors that in Grossman and Rossi-Hansberg (2008b). The schedule $t(i)$ captures the heterogeneity of these costs across tasks. We index tasks so that $t'(i) > 0$. Tasks with low indexes are those for which instructions can be communicated internationally with little loss of information. In contrast, remote performance of tasks with high indexes is problematic, because these tasks must be monitored closely by headquarters and require intensive interaction between managers and workers. The parameter β is a technological parameter that we will use in our comparative statics to model improvements in communication technology and other technological advances that facilitate coordination of activities at a distance.

For each task, we seek a sub-game perfect equilibrium in the location game. We shall find that, for some tasks and some parameter values, certain configurations of market participants after investments in outsourcing capability and choices of tentative locations imply the non-existence of pure-strategy equilibrium in the subsequent stage game. We assume that participants dislike such outcomes and so avoid these configurations along the equilibrium path.⁹ We also invoke coalition-proofness as a refinement of the set of sub-game perfect equilibria.¹⁰

⁹ Alternatively, and perhaps more convincingly, we could allow for mixed strategies in prices and final locations for configurations that do not admit a pure-strategy equilibrium. We find that the mixed strategy equilibria are difficult to characterize, but some special cases with a finite number of producers imply that there exists a range of values for $\phi/(\lambda - 1)$ that support the task locations that we study here as the unique outcomes.

¹⁰ An equilibrium is coalition proof if no group of firms can jointly change their actions at some stage in such a way that all members of the group benefit given the actions of non-members and that no member of the group has

The equilibrium of the location game determines the country in which each task is performed by each firm, and the corresponding cost. Firms mark up their per-unit costs of producing goods to maximize profits. The general equilibrium determines the measure of producers in each country, the outputs and prices of all varieties of the final good, and the factor prices in each country. In the following sections, we lay out the equilibrium conditions, beginning with those that guide the siting of a given task i .

2.1 Location of Tasks

Firms take the wage rates and aggregate output levels in the two countries as given. They decide whether to invest in outsourcing capability and where to install their capacity to perform each task, first tentatively then permanently. Later, they perform a task for themselves or subcontract with another firm depending on whether the lowest quoted price for outsourcing (if any) exceeds or falls short of the cost they would bear by performing the task in-house.

Along the equilibrium path, no firm has any incentive to invest in outsourcing capability. Such investments are costly and when firms choose their equilibrium locations, none has any advantage over its rivals that would allow it to recoup these costs. So we will begin by investigating firms' locational choices when they anticipate an absence of external suppliers.

Let's hypothesize that all firms tentatively and permanently locate a task i in East and ask whether any firm or coalition of firms has reason to locate differently without investing in outsourcing capability. If some firms have an incentive to do so, it will be those headquartered in West, because these firms must bear an offshoring cost when performing the task abroad. They face a trade-off, however, inasmuch as the savings in offshoring costs would come at the expense of scale economies. By choosing East as its location for task i , a Western producer anticipates a per-unit cost for the task of $\beta t(i)w/A(nx + n^*x^*)$, where w is the wage of a production worker in East, n and n^* are the measures of producers in East and West, respectively, and x and x^* are the respective per-firm outputs. A choice of West, if matched by other Western firms, would generate a per-unit cost of $w^*/A(n^*x^*)$. The deviation is profitable for the Western firms if and only if $\beta t(i)w/A(nx + n^*x^*) > w^*/A(n^*x^*)$.¹¹ The hypothesized location of task i in East therefore requires $i \leq I$, where I is defined by¹²

$$\beta t(I) = \frac{w^* A(nx + n^*x^*)}{w A(n^*x^*)}. \quad (1)$$

Equation (1) provides a limit on what tasks can be concentrated in East. For $i \leq I$, the

a further incentive to deviate from the altered action given the altered actions of all other group members besides itself. In our context, the group deviations will involve all firms with a common nationality switching their location for a task from one country to the other.

¹¹Note that all Western firms benefit from the switch when any one does, and none has any incentive to maintain its capacity for task i in East if the others switch to West.

¹²We have invoked coalition-proofness to make this argument. Note, however, that a tentative and permanent concentration of task i in East would be susceptible to a deviant firm that invests in outsourcing capability, locates in West, and quotes a price just below $\beta t(i)w/A(nx + n^*x^*)$. Therefore, the refinement of coalition-proofness is not required to eliminate the possibility of an equilibrium with task i concentrated in East when $i > I$.

offshoring costs are not so high as to outweigh the benefits of the scale economies and the associated benefit or cost of any difference in wages. Even if all Western firms were to coordinate a move of task i from East to West, the cost savings would not suffice to offset the loss in productivity from performing task i at smaller scale. But if $i > I$, such a coordinated move would be in the Western firms' individual and joint interest. By a similar argument, we can rule out, for $i \leq I$, an equilibrium in which all firms locate their capacity for task i in their native country. Such a location would induce a deviation by the Western firms, who would prefer to site the task in East to gain the benefits of greater scale. Our discussion presumes the existence of a task with index between zero and one such that the two alternatives of concentrated performance in East and dispersed performance are equally costly for Western firms. If no task can be concentrated in East without threat that the Western firms will switch their location to avoid the offshoring costs, then we assign $I = 0$. If all tasks are immune to profitable deviation of this sort, we assign $I = 1$.

An analogous condition applies to concentration of task i in West. Then, Eastern firms might benefit from having the task performed closer to their national headquarters, which would conserve on their offshoring costs. A potential equilibrium with all firms performing task i in West—in which Eastern firms would face a per-unit cost of $\beta t(i)w^*/A(nx + n^*x^*)$ —might be undermined by a deviation by the Eastern firms, who could perform the task near their headquarters at cost $w/A(nx)$. Such a deviation would be profitable if $\beta t(i)w^*/A(nx + n^*x^*) > w/A(nx)$.¹³ This determines another marginal task, I^* , defined by

$$\beta t(I^*) = \frac{w}{w^*} \frac{A(nx + n^*x^*)}{A(nx)}, \quad (2)$$

such that task i can be concentrated in West only if $i \leq I^*$ and can be performed locally by firms in both countries only if $i > I^*$. Again, we set $I^* = 0$ if the prescribed deviation always is profitable for Eastern firms and $I^* = 1$ if it is never so.

Taking stock, we have shown thus far that for any task $i > \max[I, I^*]$, the only candidate for equilibrium is one with local production by all firms and no task trade, as the high cost of offshoring discourages the realization of scale economies for these tasks. For any task $i \leq \min[I, I^*]$, concentration in either country remains a possible outcome, insofar as no firm or group of firms has an incentive to deviate without investing in outsourcing capability. Finally, for tasks with indexes between $\min[I, I^*]$ and $\max[I, I^*]$, there is one candidate equilibrium, since these tasks cannot be dispersed and cannot be concentrated in one of the two countries.

Let's consider further the tasks with low offshoring costs; i.e., those for which $i \leq \min[I, I^*]$. Might the potential for outsourcing discipline their location? The answer is affirmative, provided the cost of outsourcing capability is relatively small compared to the switching costs. Suppose that all firms but one tentatively locate task i in East and decline to invest in outsourcing capability. Now let a deviant locate in West and pay the small extra cost that allows it to serve others. If

¹³ Again, the joint deviation is not necessary inasmuch as a single deviant could set up capacity in East and invest in the capability to supply other Eastern firms, so as to upset an allocation with all capacity for task i concentrated in West.

the deviant anticipates that the other firms will choose East as their final location despite its own behavior, it can offer the Eastern firms a price slightly less than $w/A(nx + n^*x^*)$ and the Western firms a price slightly less than $\beta t(i)w/A(nx + n^*x^*)$. At these prices, all firms will find the option to purchase task i from the deviant firm to be more attractive than in-house production. Assuming no response from the others, the deviant would earn by this strategy profits equal to

$$\pi_d(i) \equiv \left[\frac{w}{A(nx + n^*x^*)} - \frac{\beta t(i)w^*}{A(nx + n^*x^*)} \right] nx + \left[\frac{\beta t(i)w}{A(nx + n^*x^*)} - \frac{w^*}{A(nx + n^*x^*)} \right] n^*x^*,$$

where the first term is the profit (positive or negative) that the deviant would make by performing task i for firms headquartered in East and the second term is the profit that the deviant would make by performing task i for firms headquartered in West. If $\pi_d(i) < 0$, the hypothesized concentration of task i in East is immune to this deviation. But if $\pi_d(i) > 0$, the firms tentatively located in East must formulate a best response. By retaining their capacity for task i in East, the Western firms face the price $\beta t(i)w/A(nx + n^*x^*)$. A switch of location to West would promise a production cost no greater than $\lambda w^*/A(nx + n^*x^*)$, considering that the Eastern firms would surely buy from the provider at the offered price of $w/A(nx + n^*x^*)$. So, the Western firms have incentive to switch their locations for task i . Anticipating this, the deviant firm would not set prices that presume that its customers are located in East, but instead would quote the prices $\lambda \beta t(i)w^*/A(nx + n^*x^*)$ and $\lambda w^*/A(nx + n^*x^*)$ for Eastern and Western firms that presume a switch of location. The Western firms buy from the deviant, because they can do no better producing for themselves. Given that, the Eastern firms would face a small scale of production were they to serve themselves from the East. The fact that $i \leq I^*$ implies that they too prefer to buy from the deviant despite the included offshoring cost. The deviant's profits at these prices are $(\lambda - 1)w^*[\beta t(i)nx + n^*x^*]/A(nx + n^*x^*)$, which exceed the cost of the investment if $\phi/(\lambda - 1)$ is sufficiently small.

Note that if $\pi_d(i) > 0$, task i is more efficiently performed in West, while if $\pi_d(i) < 0$, task i is more efficiently performed in East. So an analogous argument establishes the existence of a profitable deviation for task i , under similar conditions on $\phi/(\lambda - 1)$, when all firms tentatively locate in West and $\pi_d(i) < 0$. Then, the deviant locates in East with an eye toward the cost savings it can achieve by performing the task for all producers there. The deviant's action induces a switch of location for firms headquartered in East, therefore also for those headquartered in West, and the deviant's profits ultimately come from its (small) productivity advantage over those that have switched locations. Such a deviation is not profitable when $\pi_d(i) > 0$.

We define a task J as the solution to

$$\beta t(J) = \frac{wnx - w^*n^*x^*}{w^*nx - wn^*x^*}, \quad (3)$$

if the solution falls between 0 and 1, and assign $J = 0$ if $\pi_d(i) < 0$ for all i and $J = 1$ if $\pi_d(i) > 0$ for all i . If $J \in (0, 1)$, then $\pi_d(J) = 0$. Now suppose that West has a lower wage than East

in equilibrium. In the appendix, we show that one of two statements must be true.¹⁴ Either (i) $\pi_d(i) > 0$ or $\pi_d(i) < 0$ for all i , or (ii) $J > 0$ and $\pi_d(i) > 0$ for $i < J$ while $\pi_d(i) < 0$ for $i > J$.

The potential for outsourcing pins down the location of tasks with $i \leq \min[I, I^*]$. In this range, if $i \leq J \leq \min[I, I^*]$ and $w^* < w$, then the unique sub-game perfect equilibrium has task i concentrated in West.¹⁵ If $J < i \leq \min[I, I^*]$ and $w^* < w$, then the unique equilibrium has task i concentrated in East. These statements are reversed for $w^* > w$; for example, in this case $i \leq J \leq \min[I, I^*]$ implies concentration of task i in East.

We return to the tasks with intermediate offshoring costs such that $\min[I, I^*] < i < \max[I, I^*]$. We have observed that such tasks cannot be dispersed and cannot be concentrated in one of the two countries. Might a deviant that invests in outsourcing capability upset a potential equilibrium with task i concentrated in the other country? To see what can happen, consider for example parameter values that give rise to $I^* < J$ and consider a task $i \in (I^*, J)$. This task cannot be concentrated in West in equilibrium, because Eastern firms would deviate and relocate their capacity to East. The candidate equilibrium has all firms tentatively and finally locating task i in East. Now, a deviant that invests in outsourcing capability might perceive a profit opportunity, since $i < J$ implies $\pi_d(i) > 0$. However, if the deviant quotes the prices $w/A(nx + n^*x^*)$ and $\beta t(i)w/A(nx + n^*x^*)$ for Eastern and Western firms, respectively, the Western firms will have an incentive to switch their location to West. This cuts into the profits that the deviant can make by selling to them, and the deviation remains profitable only if the deviant can also induce a switch of location by the Eastern firms. However, with $i > I^*$, the Eastern firms prefer to produce for themselves at smaller scale than to bear the cost of offshoring to West. There are no prices that the deviant can quote that generate positive profits after the induced response by other firms. In fact, the stage game that ensues after investment by a single potential supplier has no equilibrium in pure strategies. If the deviant offers a pair of prices assuming that all firms will keep their capacity for task i in East, the Western firms will switch, but the Eastern firms will not. Then the deviant prefers to set higher prices which, however, leave the Western firms with a preference for staying in East. We assume that no deviation takes place in such circumstances.¹⁶

¹⁴An analogous statement to what follows applies when East has the lower wage. We will discuss below the prospects for an equilibrium with equal wages in the two countries and the properties thereof.

¹⁵When $i \leq J \leq \min[I, I^*]$ and $w^* < w$, there exists an equilibrium in which all firms locate in West. To see this, suppose that a deviant were to invest in outsourcing capability and to locate tentatively in East. In the ensuing sub-game, the remaining firms would not switch their locations to East, because if they did so, the deviant also would switch (to West) so that it could supply the market at lesser total cost. But, anticipating this, the other firms would not wish to switch their locations. Rather, the unique Nash equilibrium in the sub-game with a deviant tentatively located in East has the deviant switching its location to West while the other firms retain their locations there, so that the deviant earns no profits to cover the fixed cost of outsourcing. Similarly, when $J < i < \min[I, I^*]$ and $w^* < w$, there exists an equilibrium with all firms located in East that is immune to deviation by a firm that invests in outsourcing capability and tentatively locates in West. Analogous arguments apply to the cases in which $w^* > w$.

¹⁶Alternatively, we might allow mixed strategies in sub-games that admit no pure-strategy equilibrium, as we noted in footnote 9. We have investigated mixed strategies following an investment in outsourcing capability and found the general case difficult to solve. In several special cases with finite numbers of firms, the expected operating profits for the deviant are positive, but significantly smaller than those available to a deviant when $i < I^* < J$. This means that, for a range of values of $\phi/(\lambda - 1)$, the deviant can discipline the choice of equilibrium for tasks with $i < I^* < J$ and yet not upset an equilibrium with production concentrated in East for tasks with $I^* < i < J$.

We can summarize the findings in this section as follows. In equilibrium, firms perform all tasks in-house and none bears the cost of outsourcing capability or of switching locations. For a range of tasks with low offshoring costs, firms in each country would rather pay the cost of offshoring to reap the benefits of scale economies than perform the task at home at smaller scale. These tasks concentrate in whichever country offers the lower aggregate (production plus offshoring) costs, because the opposite location would invite deviation by a firm that could profit by investing in outsourcing capability. For a range of tasks with intermediate offshoring costs, firms concentrate in the one country in which the foreign firms there have no incentive to relocate to their home country in order to conserve on offshoring costs at the expense of scale economies. Finally, for a range of tasks with high offshoring costs, firms find these costs too high to bear. These tasks are dispersed, as all firms perform them in the country of their headquarters. Whereas the tasks with low offshoring costs are performed in the efficient location, those with higher offshoring costs need not be. Firms make their location choices without regard to the externality it may impose on others, and a potential supplier cannot internalize the externality when all the firms of a given nationality would rather perform the task locally than reap the scale benefits of concentrated production abroad.

2.2 General Equilibrium

The remainder of the equilibrium conditions are more familiar. Firms price their products optimally in the light of the demands they face, while free entry drives profits to zero. Meanwhile, factor markets clear in each country.

Let c and c^* denote the unit cost of a typical final good in East and West, respectively, which reflect the equilibrium locations of the various tasks. Let \mathcal{E} denote the set of tasks that are concentrated in East, \mathcal{W} the set of tasks that are concentrated in West, and \mathcal{B} the set of tasks that are performed locally by firms in both countries. Of course tasks in \mathcal{E} represent offshoring for firms headquartered in West, while tasks in \mathcal{W} represent offshoring for firms headquartered in East. There is no offshoring of the tasks in \mathcal{B} . In view of the costs of offshoring and the different scales of output for tasks that are traded and non-traded, we have

$$c = \frac{wM(\mathcal{E})}{A(nx + n^*x^*)} + \frac{w^*T(\mathcal{W})}{A(nx + n^*x^*)} + \frac{wM(\mathcal{B})}{A(nx)} \quad (4)$$

and

$$c^* = \frac{wT(\mathcal{E})}{A(nx + n^*x^*)} + \frac{w^*M(\mathcal{W})}{A(nx + n^*x^*)} + \frac{w^*M(\mathcal{B})}{A(n^*x^*)}, \quad (5)$$

where $M(\mathcal{Z})$ is the Lebesgue measure of \mathcal{Z} for $\mathcal{Z} = \{\mathcal{E}, \mathcal{W}, \mathcal{B}\}$. $T(\mathcal{Z})$ is given by

$$T(\mathcal{Z}) = \int_{i \in \mathcal{Z}} \beta t(i) di$$

for $\mathcal{Z} = \mathcal{E}$ and \mathcal{W} . So $T(\mathcal{Z})/A(nx + n^*x^*)$ is the total amount of labor needed per unit of output to

perform the tasks in \mathcal{Z} for an offshore firm when labor productivity is $A(nx + n^*x^*)$.¹⁷ In (4), the three terms are the total cost to a firm headquartered in East of the tasks that are concentrated in East, the tasks that are concentrated in West, and the tasks that are dispersed, respectively. The interpretation of (5) is similar.

Given c and c^* , the firms practice mark-up pricing. This yields, via the demand functions, a relationship between relative costs of Eastern and Western firms and relative quantities produced (and consumed) of the different varieties, namely

$$\frac{x}{x^*} = \left(\frac{c}{c^*}\right)^{-\sigma}, \quad (6)$$

where, as defined before, $\sigma > 1$ denotes the elasticity of substitution between any pair of varieties. Free entry equates variable profits to fixed costs, which are sf for a firm headquartered in East and s^*f for a firm headquartered in West, where s and s^* are the salaries of managers in East and West, respectively. By familiar calculations, the zero-profit conditions imply

$$s = \frac{cx}{f(\sigma - 1)} \quad (7)$$

and

$$s^* = \frac{c^*x^*}{f(\sigma - 1)}. \quad (8)$$

Finally, we have the factor-market clearing conditions. Managers are employed only in headquarters, where they perform activities that are independent of scale. In each country f managers are needed per firm, which implies

$$nf = H \quad (9)$$

and

$$n^*f = H^*. \quad (10)$$

Workers in each country are employed in tasks that are performed locally by national firms and in affiliates of foreign firms. Tasks in \mathcal{E} do not use any Western labor and tasks in \mathcal{W} do not use any Eastern labor. Considering the demands by local and foreign firms for the tasks that are concentrated in one country and for those that are dispersed, we have

$$\frac{M(\mathcal{E})}{A(nx + n^*x^*)}nx + \frac{T(\mathcal{E})}{A(nx + n^*x^*)}n^*x^* + \frac{M(\mathcal{B})}{A(nx)}nx = L \quad (11)$$

and

$$\frac{T(\mathcal{W})}{A(nx + n^*x^*)}nx + \frac{M(\mathcal{W})}{A(nx + n^*x^*)}n^*x^* + \frac{M(\mathcal{B})}{A(n^*x^*)}n^*x^* = L^*. \quad (12)$$

¹⁷We assume \mathcal{E} , \mathcal{W} , and \mathcal{B} are elements of the Borel σ -algebra and that $t(\cdot)$ is Lebesgue measurable. For the case in which $w \neq w^*$, we will find that the sets \mathcal{E} , \mathcal{W} , and \mathcal{B} are connected intervals, so the integral that defines $T(\cdot)$ is a standard Riemann integral. If $w = w^*$, the theory imposes no structure on the sets \mathcal{E} and \mathcal{W} (\mathcal{B} is still a connected interval). In this case, we restrict attention to sets \mathcal{E} and \mathcal{W} that are elements of the Borel σ -algebra and use the Lebesgue integral. Of course, this restriction has no effect on the general equilibrium properties of our economy.

The three terms on the right-hand side of (11) are, respectively, the labor employed in Eastern firms to perform tasks that are concentrated in East, the labor employed in Eastern subsidiaries of firms based in West, and the labor employed by Eastern firms in tasks that are not traded. The interpretation of the terms in (12) is analogous.

We have not yet chosen a numeraire. Let $w^* = 1$. Then (9) and (10) determine n and n^* . Given the allocation of tasks to the sets \mathcal{E} , \mathcal{W} and \mathcal{B} and the equilibrium values of n and n^* , (4)-(6) and (11)-(12) determine c, c^*, x, x^* and w .¹⁸ Finally, (7) and (8) determine s and s^* residually. Our next task is to characterize the patterns of specialization that can emerge in equilibrium.

3 Patterns of Specialization

In this section, we explore the patterns of specialization that can emerge when there is task trade between similar countries. We use a combination of numerical and analytical methods to describe equilibrium configurations of task allocation. We begin by illustrating examples of patterns that can arise when offshoring costs are, respectively, high and low. We then provide a general result that links task allocation to relative wages and relative aggregate outputs and discuss the relationship between these endogenous variables and relative country size.

3.1 Equilibrium Allocations with High Offshoring Costs

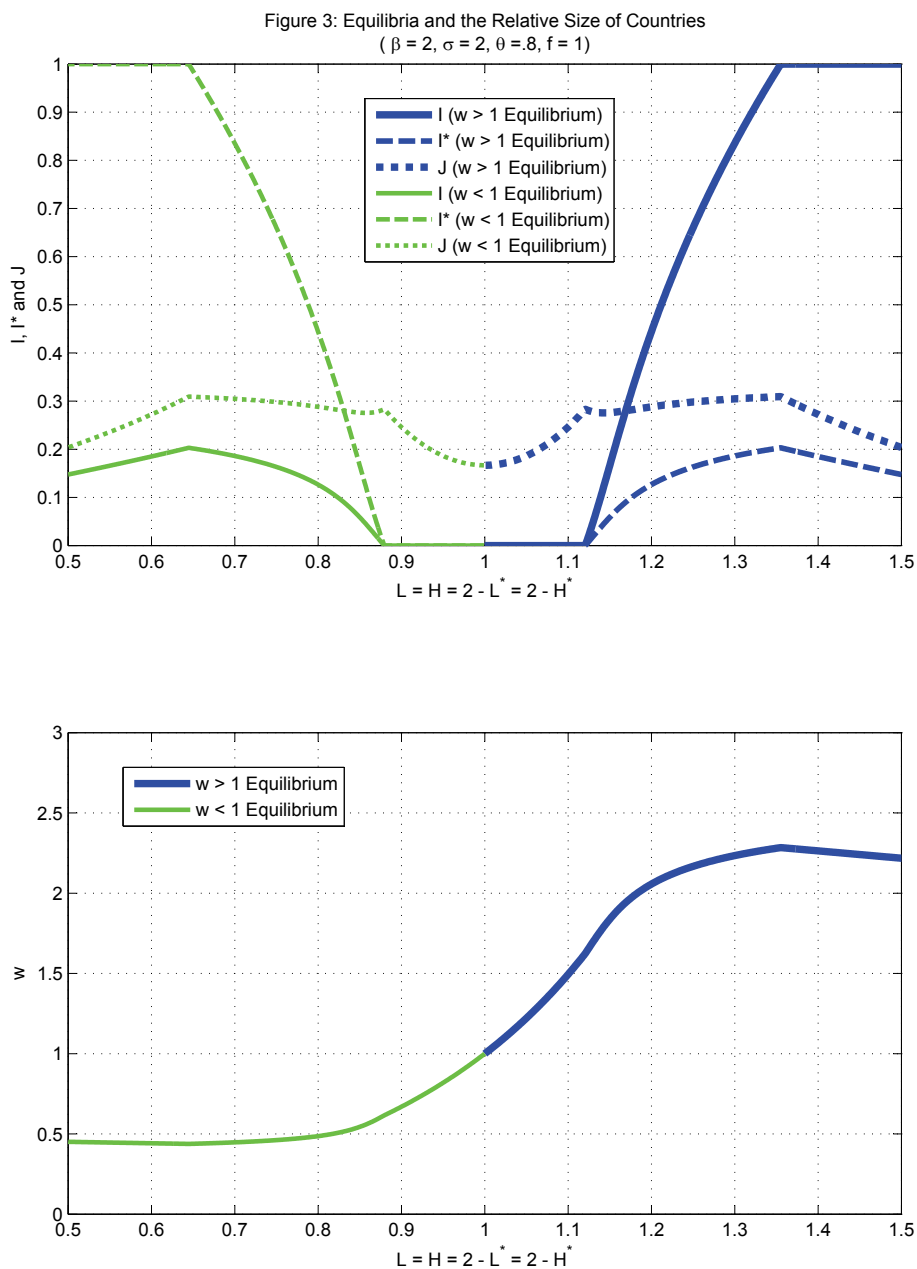
Figure 3 depicts a typical outcome when offshoring costs are reasonably high. The figure is drawn for the case of a linear offshoring-cost schedule, with $t(i) = 1 + i$ and $\beta = 2$. The external economies take the form $A(X) = X^\theta$, with $\theta = 0.8$. For other parameters, we take $\sigma = 2$, $f = 1$, $L + L^* = 2$, and $H + H^* = 2$.

The top panel of the figure shows the threshold tasks I, I^* and J for different divisions of the world's labor supply between the two countries. In all cases, we take endowments of both factors to be equal, so that $H = L$ and $H^* = L^*$. Using the limits for profitable deviations, we can describe the equilibrium allocation of tasks across countries. The bottom panel shows the corresponding relative wage; recall that the foreign wage serves as numeraire. For ease of interpretation, we distinguish visually the outcomes with $w > 1$ from those with $w < 1$; the former are depicted with thick, dark curves, the latter with curves that are thinner and lighter in shade.

The figure shows that when offshoring costs are high and the world's resources are divided almost evenly between the two countries, $I = I^* = 0$. For a range of values of L on either side of unity, concentration of any task in some country would be undermined by a deviant offering to perform that task for firms headquartered in the other country. The deviant could profit by avoiding the high offshoring costs. In the unique equilibrium that arises for sufficiently high β and L close to L^* , no offshoring takes place. As the bottom panel shows, the larger country enjoys

¹⁸The solution to these five equations for given \mathcal{E} , \mathcal{W} and \mathcal{B} is unique. However, the sets \mathcal{E} , \mathcal{W} and \mathcal{B} are themselves determined by the equilibrium values of the other variables. This suggests the possibility of multiple equilibria, which we discuss further below.

the relatively higher wage, as its superior scale gives it a productivity advantage in performing all tasks.



For a given value of β , the incentives to offshore grow as the countries become more unequal in size. When one country is substantially larger than the other, firms in the smaller country have much to gain by performing some tasks abroad. The productivity gains associated with the scale economies outweigh the relatively high costs of offshoring. Yet it may not be possible for task trade to flow only in one direction. If firms headquartered in the small country offshore tasks to the large country, their foreign subsidiaries use resources that otherwise would be employed by local firms.

An incipient excess demand for the large country's workers puts upward pressure on the wage there, which in turn creates an incentive for firms headquartered in the large country to perform tasks in the lower-wage country.

Consider circumstances in which East is substantially larger than West.¹⁹ We see in the figure that when L is sufficiently greater than L^* , $I > I^* > 0$ and $J > I^*$. The tasks with $i \leq I^*$ can be concentrated in either country without risk that national producers of the other country will be drawn home by a deviant supplier. These tasks are more efficiently concentrated in West than East, because the wage in West is lower and the burden of offshoring these tasks is not so great. The next range of tasks has $I^* \leq i < I$. Some or all of these tasks may be more cheaply concentrated in West than in East.²⁰ However, the siting of these tasks in West is undermined by potential deviation by the Eastern firms, who would prefer to locate and produce for themselves in East. Therefore, the tasks in this range are performed only in East, where the large scale of output makes up for the moderately high cost of offshoring borne by Western firms. Finally, for tasks with $i > I$, even specialization by the high-wage country is not sustainable, inasmuch as a coalition of firms from the low-wage country would like to produce the task back home. These tasks that are most costly to offshore are performed locally in both countries in the general equilibrium.

Notice that, for all L and L^* depicted in Figure 3, if offshoring occurs at all, task trade flows in both directions. This is a consequence of the particular parameter values used in the figure, specifically our choice of $\sigma = 2$.²¹ However, one feature of the equilibria depicted in the figure is more general. For all values of L and L^* , if a task i is concentrated in the low-wage country and another task i' is concentrated in the high-wage country, then $i' > i$. We will see in Section 3.3 that if wages in the two countries are not equal, the allocation of tasks always obeys this rule: tasks that are performed only in the high-wage country bear higher offshoring costs than those that are performed only in the low-wage country.

The figure shows the larger country enjoying the higher wage. Its wage advantage derives from two sources. First, as before, the larger country benefits from having greater scale in the tasks that are performed locally in both countries. But now the larger country benefits as well from the allocation of the traded tasks. In this allocation, the smaller country performs locally the tasks that are easiest to offshore, while the larger country performs locally tasks that are more costly to offshore. This redounds to the benefit of the larger country's factors of production. For all parameter values that we have examined, there exists an equilibrium in which the larger country has the higher wage. But, as we shall see in the next section, there can sometimes exist a second equilibrium in which the smaller country enjoys the higher wage. In such circumstances, it remains true that the low-wage country captures the tasks that are easiest to offshore and the high-wage country captures those that are more difficult to offshore.

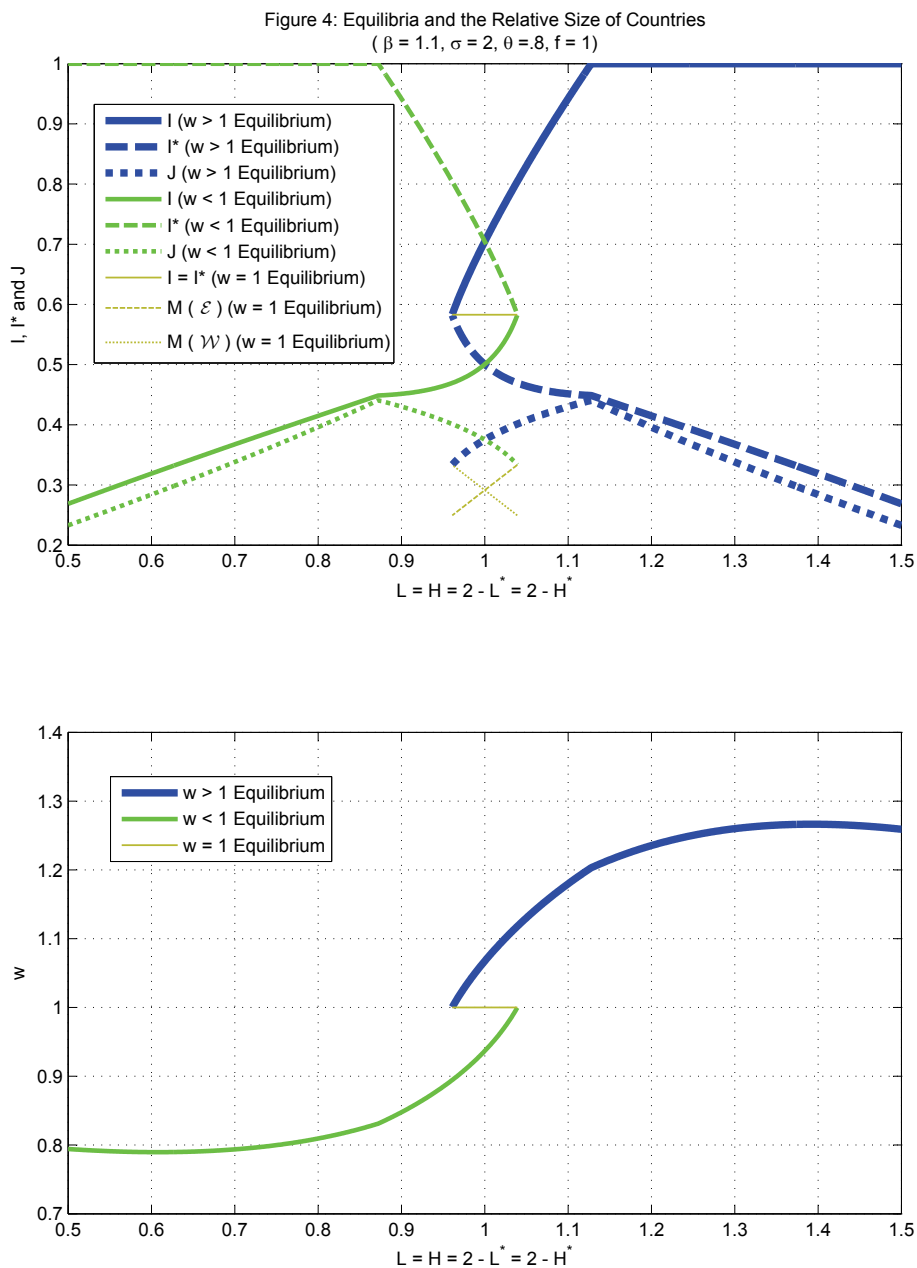
¹⁹Notice the symmetry of the figure. Everything that we say about equilibria with $L > 1$ applies as well to equilibria with $L < 1$, with the names of the countries reversed.

²⁰The tasks below the thick dotted line can be performed at lower total cost in the low-wage West than in high-wage East; those above the dotted line can be done at lower total cost in East.

²¹When $\sigma = 2$ and $A(X) = X^\theta$, $I = 0$ if and only if $I^* = 0$. This property of Figure 3 does not apply for other values of σ or for other forms of the externality function when $\sigma = 2$.

3.2 Equilibrium Allocations with Low Offshoring Costs

Figure 4 is drawn for similar parameter values as Figure 3, except that $\beta = 1.1$. The figure depicts three equilibria that exist when resources are almost evenly divided between the two countries. For L significantly greater than L^* (or *vice versa*) the equilibrium is unique.



The various equilibria in Figure 4 are distinguished by the thickness and shading of the curves. Consider the three curves in the top panel that are thickest and darkest and look first at the portions of these curves that apply for $L \geq L^*$. The curves describe an equilibrium with $I > I^* > J > 0$. Tasks with i less than J are most cheaply performed in West and no profitable deviation prevents

them from concentrating there. Tasks with i between J and I are most cheaply performed in East and again nothing prevents them from locating there. Finally, tasks with $i > I$ cannot be concentrated, because a deviant supplier in either country could attract the business of local producers. Notice that for L sufficiently large, $I = 1$, which means that no tasks are performed in both countries. This reflects the modest cost of offshoring for the parameters used to draw the figure. Of course, if $t(i)$ were not linear but instead rose rapidly with i , then the equilibrium would always feature some non-traded tasks. The bottom panel of Figure 4 again shows the associated relative wage. The thick curve corresponding to the equilibrium just described has $w > 1$ for all $L \geq L^*$. The larger country has the higher wage for the reasons discussed at the end of Section 3.1.

But notice that the thick curves do not begin at $L = L^* = 1$. In other words, we can have an equilibrium in which the *smaller* country has the higher wage and exports the tasks that have intermediate offshoring costs. For L slightly smaller than L^* and $w > 1$, the ordering of the boundary values is $I > I^* > J$ as before. Again, West alone performs the tasks with $i \leq J$, East alone performs the tasks with $J < i \leq I$, and both countries perform tasks with $i > I$. Here, East generates greater aggregate output than West (i.e., $nx > n^*x^*$) despite its smaller size and correspondingly smaller endowment of managers. The shortfall in the number of its firms compared to West ($n < n^*$ due to $H < H^*$) is more than made up by greater sales per firm ($x > x^*$). Because the East has a greater scale of output, it enjoys a productivity advantage in the tasks that are performed locally by all firms. It also benefits by capturing the tasks that are more difficult to offshore among those that are traded. Its overall cost advantage ($c < c^*$) underlies its superior sales per variety, which in turn justifies its higher wage and the pattern of specialization.

The requirement for the East to perform the tasks with intermediate offshoring costs for producers worldwide strains its small resource base. If L is sufficiently smaller than L^* , the East lacks the workers it would need to perform a sufficient range of tasks that are relatively costly to offshore, and then its costs would not be low enough to justify its larger scale and higher wage. In such circumstances, an equilibrium in which the smaller country has the higher wage and the higher aggregate output does not exist. For the parameters used to draw Figure 4, an equilibrium with $w > 1$ exists for $L > .96$, but not otherwise.

The thick curves in Figure 4 are analogous to the thick curves in Figure 3. In both cases, the thickness refers to the fact that $w > 1$; i.e., the wage in East is greater than the wage in West. Recall that, in Figure 3, the thick curves incorporate a range of values of L for which $w > 1$ and $I = I^* = 0$. As the offshoring costs fall, this range of values shrinks and eventually disappears. Once these costs are low enough that offshoring takes place even when $L = L^*$, the outcome looks qualitatively like that in Figure 4 instead of that in Figure 3.

The curves of medium shade and thickness depict a second equilibrium, analogous to the similarly shaded curves in Figure 3. Notice how they reflect the thicker curves across the vertical at $L = 1$; i.e., an equilibrium with $L < L^*$ corresponds to one with $L > L^*$, except for the reversal of country names. For L slightly greater than L^* , the curves of medium thickness represent an equilibrium in which the wage is higher in West despite its (slightly) smaller size and the tasks with

intermediate trading costs are concentrated there. This equilibrium exists for $L > L^*$ for much the same reason as does the thick equilibrium with $w > 1$ when $L < L^*$. For L sufficiently larger than L^* , an equilibrium with $w < 1$ cannot be sustained.

Finally, the figure depicts a third equilibrium that exists for exactly the same range of L and L^* that admits the coexistence of a thick equilibrium with $w > 1$ and a medium-thick equilibrium with $w < 1$. The equilibrium represented by the thin curves features equal wages in the two countries.

If wages are the same in the two countries, all task can be performed at lesser aggregate cost in the country that has the larger scale of aggregate production. No task could be concentrated in the country with the smaller aggregate output, because such an outcome would be undermined by a price-discriminating supplier locating in the country with greater output. But with countries of similar size, if all traded tasks were concentrated in the same country, the two labor markets could not clear. It follows that an equilibrium with equal wages must have equal aggregate outputs as well; that is, $nx = n^*x^*$.

With wages and aggregate outputs equalized, there is nothing to determine the siting of any task for which specialization is viable. Nonetheless, the unit cost equations (4) and (5) and the labor market clearing conditions (11) and (12) determine the measures of traded tasks that are performed in each country and the aggregate offshoring costs borne by producers of either nationality. Also, with $w = w^*$ and $nx = n^*x^*$, the incentives for a deviant supplier to upset an equilibrium with concentrated task performance are the same for both countries. Therefore, $I = I^*$ and this common value represents the boundary between traded and non-traded tasks. The figure shows $M(\mathcal{E})$ and $M(\mathcal{W})$ for the equal-wage equilibrium, as well as $I = I^*$.

Notice that $M(\mathcal{W})$ meets up with the J curve for the thick equilibrium at its left-most extreme, while $M(\mathcal{E})$ meets up with the J curve for the medium-thick equilibrium at its right-most extreme. These are also the values of L and L^* at which the relative wages converge to one in the thick and medium-thick equilibria, respectively. In other words, the equal-wage equilibrium and the thick equilibrium converge as the relative wage approaches one (from above) in the latter. Similarly, the equal-wage equilibrium and the medium-thick equilibrium converge as the relative wage approaches one (from below) in the latter.

Although the equal-wage equilibrium has an indeterminate pattern of specialization, there are two constraints on the allocation of the traded tasks. First, an equilibrium allocation must satisfy $T(\mathcal{E}) \geq T([0, M(\mathcal{E})])$, because the total offshoring costs for tasks concentrated in East must be at least the cost of offshoring the measure $M(\mathcal{E})$ of tasks that are least costly to offshore. Second, the allocation of tasks must obey $T(\mathcal{E}) \leq T([0, I]) - T([0, M(\mathcal{W})])$, because the offshoring costs for tasks concentrated in East can be at most the cost of offshoring the measure $M(\mathcal{E})$ of traded tasks that are most costly to offshore; i.e., it is maximized when the measure $M(\mathcal{W})$ of tasks with the lowest offshoring costs locate in West. An equilibrium with equal wages in which the measure $M(\mathcal{E})$ of tasks that are least costly to offshore is concentrated in East is identical to the limiting equilibrium with $w < 1$ as $w \rightarrow 1$. And the equilibrium with equal wages in which the measure $M(\mathcal{E})$ of tasks that are most costly to offshore is concentrated in East is identical to the limiting

equilibrium with $w > 1$ as $w \rightarrow 1$. This explains the convergence of the various thin and thicker curves in Figure 4. When the gap between L and L^* grows too large, one of the constraints must be violated, and so the equal-wage equilibrium ceases to exist.

We offer one further observation about the equal-wage equilibrium. Although our model lacks explicit dynamics, the equal-wage equilibrium has a knife-edge property that suggests instability under plausible adjustment mechanisms. Suppose we perturb such an equilibrium by misallocating a few tasks in such a way that total production costs in the two countries remain unchanged. Then the labor markets will fail to clear, which will exert pressure on the relative wage. As soon as the wage equality is broken, the remaining traded tasks will relocate so that those with low offshoring costs are concentrated in the low-wage country and those with intermediate offshoring costs are concentrated in the high-wage country. In other words, a small perturbation creates incentives for a large reallocation of resources and moves the economy into the neighborhood of one of the two equilibria with unequal wages. For this reason, we do not consider further the equal-wage equilibrium in the remainder of this paper.

3.3 Task Allocation: A General Result

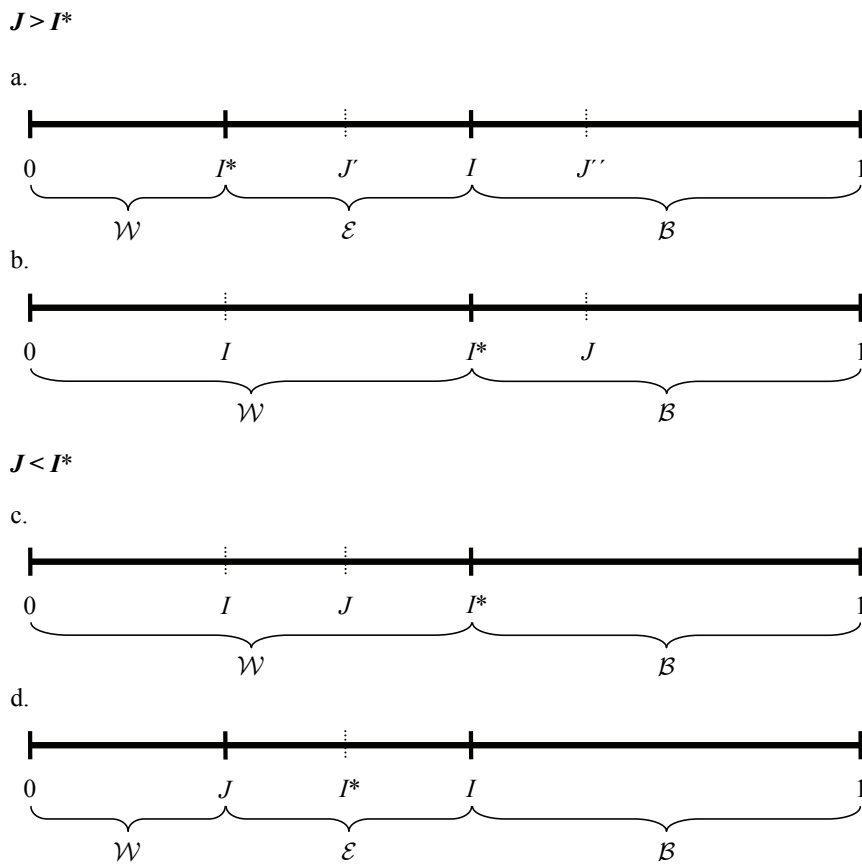
Let us summarize the lessons from the two examples. First, if task trade is relatively costly, the general equilibrium may be unique. In the extreme, all tasks are performed locally and the potential benefits from specialization are foreclosed. But when countries differ greatly in size, specialization can occur even with reasonably high costs of offshoring. Producers in a very small country will pay a sizeable offshoring cost to reap the benefits of scale economies. As they do so, the decline in relative demand for labor in their country spells a reduction in the country's relative wage. This in turn induces firms based in the large country to relocate tasks to profit from the cheaper labor abroad. As the technology for communication and coordination improves, the endowment gap necessary for offshoring to take place shrinks. When offshoring costs are sufficiently low, task trade occurs between countries of similar size and multiple equilibria can exist. The country with the higher wage produces more aggregate output of final goods and performs the traded tasks that are more difficult to offshore. This country thereby enjoys a cost advantage that validates its higher wage and greater sales per firm. There will be one equilibrium in which the larger country captures the higher wage and, if the size disparity is not too great, another in which the smaller country captures the higher wage. When these two equilibria are present there exists a third equilibrium with equal wages, but this one has knife-edge properties that suggest instability.

The equilibria with unequal wages share a common pattern of specialization. In all of them, the tasks with the lowest offshoring costs are performed only in the country with the lower wage and lesser aggregate output, while the tasks with intermediate offshoring costs are performed only in the country with the higher wage and greater aggregate output. Tasks with the highest offshoring costs are performed locally by firms in both countries. We now proceed to show that this pattern of specialization arises quite generally.

Consider any equilibrium with unequal wages in the two countries. For concreteness, let $w > 1$,

so that the wage in East exceeds that in West. Then as we have argued previously, tasks with $i > I$ cannot be concentrated in East and tasks with $i > I^*$ cannot be concentrated in West. Among those tasks with indexes below both I and I^* , those with $i > J$ are performed only in the East and those with $i \leq J$ are performed only in the West. These restrictions on where tasks can and cannot be performed have several immediate implications. First, tasks with $i < \min[I^*, J]$ are performed only in West, because no deviation by an outsourcing firm or a coalition of Eastern firms is profitable for these tasks. Second, tasks with $i > \max[I, I^*]$ are dispersed, because concentration is undermined by a deviant that serves producers of a single nationality. Third, tasks with $I^* \leq i < \max[I, I^*]$ are performed only in East, because concentration of these task is viable there, whereas the same is not true of concentration in West.

Figure 5



Panels a and b of Figure 5 depict patterns of specialization when $J > I^*$. The former has $I^* \leq I$, while the latter has $I^* > I$. In either case, the tasks with $i \leq I^*$ are performed only in West. In panel a, tasks with $I^* < i \leq I$ are performed only in East. No such range of tasks exists in panel b. Finally, in each panel, the tasks with $i > \max[I, I^*]$ are performed locally by firms in both

countries. Notice that the location of J plays no role in these circumstances; e.g., the allocation of tasks in panel a is the same for $J = J'$ or $J = J''$.

Panels c and d of Figure 5 depict the possible outcomes when $J < I^*$. The former has $I < J$ while the latter has $I > I^*$. These are the only possibilities when $J < I^*$, because the following lemma rules out any configuration with $w > 1$ and $J < I < I^*$.

Lemma 1 *If $w > 1$, then $J < I$ implies $I > I^*$.*

Proof. See the appendix. ■

When the ordering is as depicted in panel c, the tasks with $i < I < J$ cannot be concentrated in East, because such an allocation is undermined by an outsourcing firm that locates in West and target all producers. But the tasks with $i > I$ also cannot be concentrated in East, because such an allocation is undermined by a coalition of Western firms that prefers to produce the task in West. It follows that no task is concentrated in East, much like in panel b. Tasks with $i \leq I^*$ are performed only in West, while tasks with $i > I^*$ are dispersed.

When the ordering of panel d applies, tasks with $i \leq J$ are concentrated in West, where the aggregate costs of performing these tasks is minimized. Tasks with $J < i \leq I$ are concentrated in East, where again the aggregate costs of performing these tasks is minimized. Finally, tasks with $i > I$ are performed locally by all firms, because concentration would be susceptible to the threat of a profitable deviation by a firm serving Western producers.

The patterns that appear in Figure 5 apply to any equilibrium with $w > 1$.²² Moreover, the following lemma states that wages and scale go hand in hand.

Lemma 2 *$w > 1$ if and only if $nx > n^*x^*$.*

Proof. See the appendix. ■

By enumerating all of the possible orderings of I, I^* and J that can arise when $w > 1$ and investigating the pattern of specialization in each, we have established the following general result:

Proposition 1 *The pattern of specialization is characterized by*

(i) *concentrated performance of tasks with the lowest offshoring costs in the country with low wages and low aggregate output,*

(ii) *concentrated performance of tasks with intermediate offshoring costs in the country with high wages and high aggregate output, and*

(iii) *dispersed performance of tasks with the highest offshoring costs in both countries.*

²²The patterns that arise when $w < 1$ are analogous. For example, when $w < 1$ and the ordering of threshold tasks is $I < J < I^*$, then the tasks with $i \leq I$ are in \mathcal{E} , the tasks with $I < i \leq I^*$ are in \mathcal{W} , and the tasks with $I > I^*$ are in \mathcal{B} . This pattern is analogous to that in Figure 5a; the other panels have similar analogs.

The proposition does not exclude the possibility that no tasks are concentrated in one of the countries, or that no tasks are performed ubiquitously; i.e., one or more of the sets \mathcal{E} , \mathcal{W} and \mathcal{B} may be empty.

The pattern of specialization described by Proposition 1 holds intuitive appeal in the light of our previous discussion. Tasks that are very costly to offshore are performed locally, for obvious reasons. For the other tasks, firms in the country with the smaller aggregate output have the most to gain from moving tasks abroad, while those in the country with the larger aggregate output have the most to lose from the communication and coordination costs.²³ Market forces drive the tasks that are most difficult to offshore (among those that are traded) to the country with the larger aggregate output to reap the cost savings. In the process, the wage there is bid up, creating incentives for firms in the high-output country to offshore tasks that can readily be moved to the low-wage location. Moreover, the pattern of specialization conforms with global efficiency as concerns the location of traded tasks; those with high offshoring costs are concentrated in the country with greater aggregate output to conserve on transactions costs. Those with low offshoring costs are concentrated in the country with lesser aggregate output to conserve on resource use. But the extensive margin of offshoring is not efficient, because a deviant can induce a group of producers to source near their national headquarters without regard for the adverse effect on the productivity of firms headquartered elsewhere.

4 Comparative Statics

In this section, we explore the relationship between the equilibrium outcomes and some of the key parameters of the model. In particular, we focus on the connection between the pattern of specialization – as revealed by the threshold tasks I , I^* and J – and the extent of increasing returns to scale, the extent of product differentiation, and the size of offshoring costs. Our main purpose is to gain a better understanding of how the pattern and volume of task trade are determined in the model, along with the implications for relative wages.

Our analysis uses numerical methods. We present only a few examples which, however, are illustrative of our findings for an extensive exploration of the parameter space. We assume throughout that the spillover function takes the form $A(X) = X^\theta$, with $\theta < 1$ as required for concavity. Also, offshoring costs are given by $\beta(1 + i)$ for $i \in [0, 1]$. For the most part, we present results for parameters that imply zero task trade between countries of similar size, so that the equilibrium for all values of L and L^* is unique.

4.1 Extent of Increasing Returns to Scale

We begin with the strength of external economies, as captured by θ . When $\theta = 0$, productivity is constant and independent of the scale at which a task is performed. As θ grows larger, returns to

²³Aggregate output need not correspond to country size, as the smaller country may produce more per brand if its lower costs generate greater demand. The equilibrium in Figure 4 in which the smaller country has higher wages is one in which its aggregate output of final goods exceeds that in the larger country.

scale increase. We restrict attention to spillover functions with $\theta < 1$, because a function with an elasticity greater than one would allow unbounded output with finite resources.

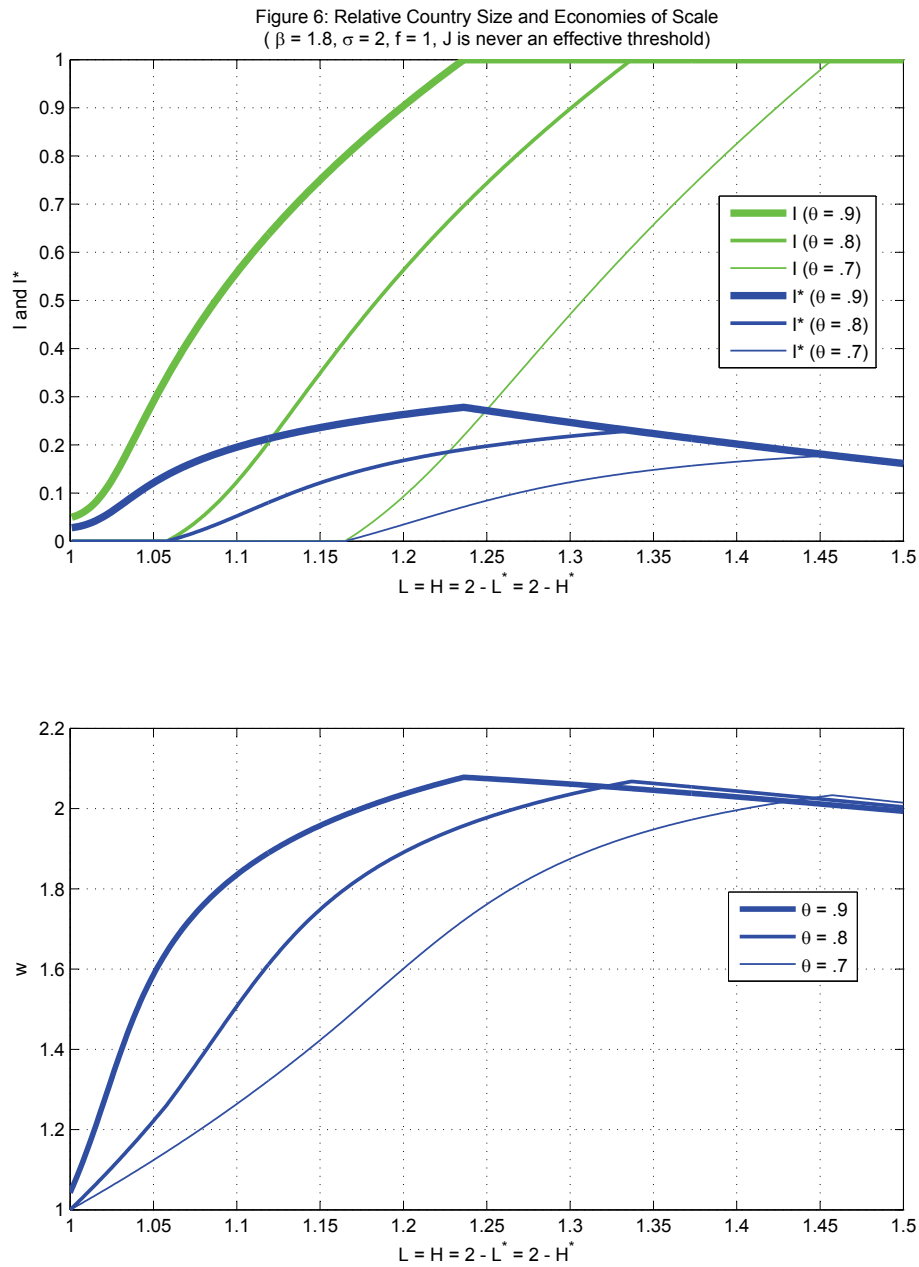


Figure 6 shows equilibria for three different values of θ . The computations that underlie this figure use $\beta = 1.8, \sigma = 2, f = 1$ and $L + L^* = H + H^* = 2$. For these parameter values, the effective determinants of task allocation are the threshold values I and I^* , which identify the tasks that can be concentrated in each country without being susceptible to deviation by a firm that locates in the other and attracts local firms there as customers. We suppress the J curves to minimize clutter, as the value of J has no bearing on the equilibria that are depicted. In all these cases, tasks with

$i \leq I^*$ (if any) are concentrated in the low-wage West, tasks with $I^* < i \leq I$ are concentrated in the high-wage East, and tasks with $i > I$ (if any) are dispersed.²⁴

The top panel in Figure 6 shows that I increases with θ for all values of L as long as $I < 1$. When some tasks are performed locally in both countries, stronger scale economies mean greater incentives for concentration of task performance and so a smaller set of dispersed tasks. The set of dispersed tasks also shrinks as the gap in resource endowments grows, for reasons that we discussed in Section 3.1.

In the figure, both I^* and $I - I^*$ grow monotonically with θ for all values of L and θ such that $I < 1$. That is, as long as there are some tasks that are performed locally in both countries, a strengthening of scale economies expands the measure of tasks that is concentrated in each one.²⁵ In such circumstances, an increase in θ also increases the relative wage of the high-wage country, as can be seen in the bottom panel of the figure. As the set of tasks that is concentrated in each country grows, the demand for Eastern labor grows by more than the demand for Western labor, because the East is performing tasks that are more costly to offshore than those performed in the West. Moreover, an increase in θ causes the set \mathcal{E} to shift to the right and thus makes the tasks concentrated in East even more labor demanding relative to those concentrated in West than before. The growth in the relative demand for Eastern labor exerts upward pressure on its relative wage.

Once $I = 1$, however, this mechanism —of expansion in I that causes an increase in relative demand for Eastern labor— can no longer operate. Then a further rise in θ has no effect on I^* . The impact effect of this further strengthening of scale economies is to enhance the incentives for concentration in West. But any expansion of the set of tasks performed in West exerts upward pressure on its relative wage, which dims the enthusiasm of the Eastern firms for performing the marginal tasks abroad. In the end, aggregate outputs rise in both countries by the same percentage as the increase in labor productivity, the pattern of specialization remains unchanged, and the relative wage of the East falls (see the bottom panel of Figure 6, where the fall in w as θ increases from 0.7 to 0.9 is slight, but visible).

4.2 Extent of Product Differentiation

The extent of product differentiation is captured by the elasticity of substitution, σ . We need at least a unitary elasticity of substitution between goods for profit-maximizing prices to be finite. As the final goods become closer substitutes, any cost advantage that producers in one country enjoy relative to producers in the other results in a larger ratio of sales per firm; see equation (6). Recall that producers based in the high-wage country have lower per unit costs as a result of their greater

²⁴For $\theta = 0.9$, we find $I > 0$, $I^* > 0$, and $w > 1$ when the countries are identical in size ($L = L^* = H = H^*$). The existence of this asymmetric equilibrium with positive task trade and unequal wages when the countries are identical in size implies that there exist two other equilibria when $\theta = 0.9$ and L is sufficiently close to L^* , one with $w = 1$ and another with $w < 1$. We do not show these equilibria in Figure 6.

²⁵At given wages, an increase in θ dampens profitability for a deviant that would sell to only one set of national producers and therefore both I and I^* rise at the initial relative wage.

productivity in tasks that are performed locally in both countries and their country's specialization in tasks that have relatively greater offshoring costs. Thus, an increase in σ magnifies the scale advantage of the high-wage country.

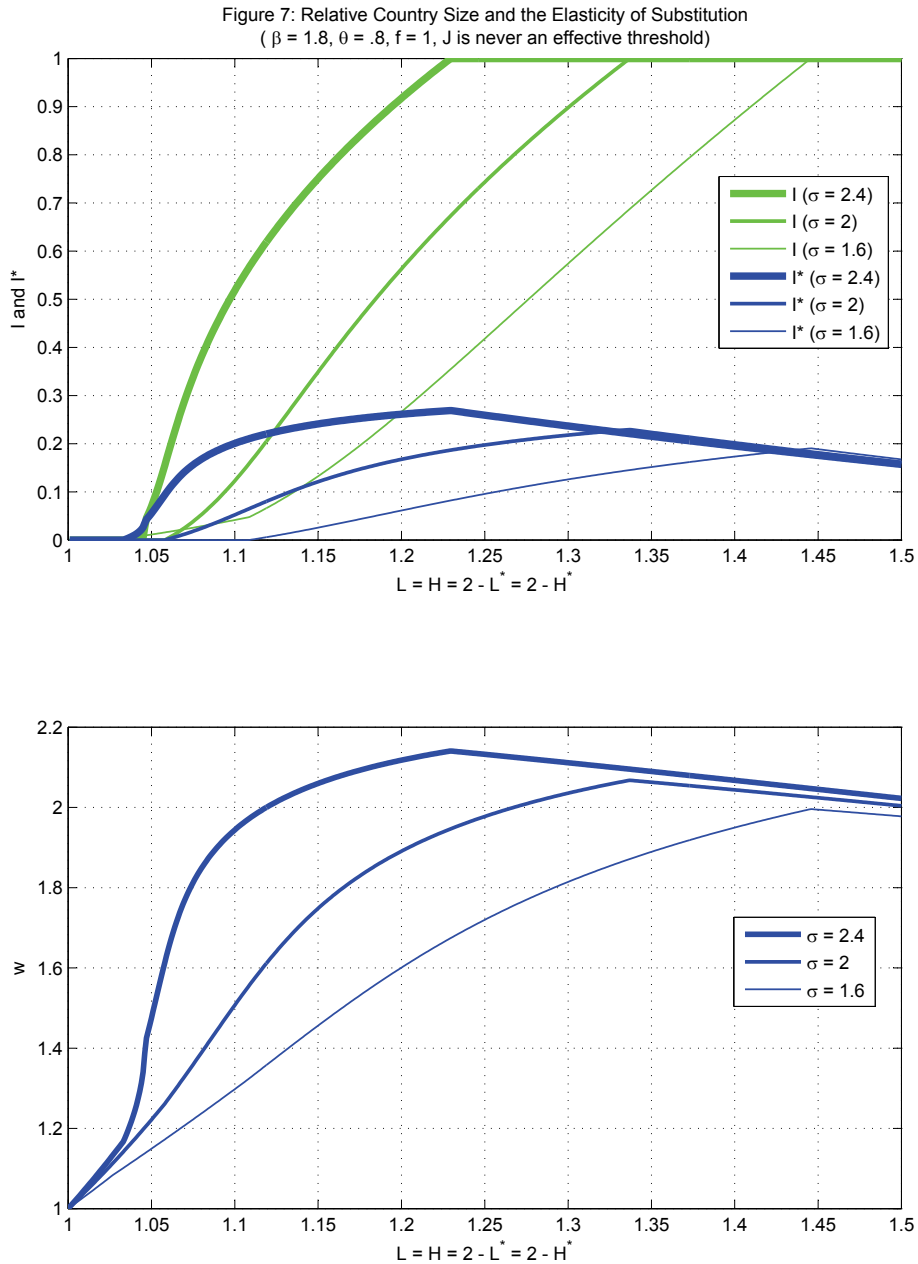


Figure 7 uses the same parameter values as Figure 6, except that we fix θ at 0.8 and consider three values of σ ranging from 1.6 to 2.4. Again, the values of I and I^* determine the equilibrium allocation of tasks, so we suppress the J curves. Although it is difficult to see in the figure, we find that if $\sigma > 2$, there exists a range of country sizes for which only firms in the West perform tasks offshore ($I^* > I = 0$), while if $\sigma < 2$, there is a range of country sizes for which only firms

in the East perform tasks offshore ($I > I^* = 0$). But, in either case, if the countries are sufficiently different in size, task trade flows in both directions. As we have noted previously, the case of $\sigma = 2$ is special, because it has tasks concentrated in both countries whenever any tasks are concentrated in one.

Consider first a division of resources such as $L = 1.15$, for which $1 > I > I^* > 0$; i.e., the sets \mathcal{W} , \mathcal{E} and \mathcal{B} are all non-empty. The relative cost advantage of Eastern firms that derives from the allocation of traded tasks results in a larger ratio of sales per firm as substitutability increases. As a result of this and the fact that the numbers of firms are fixed by the endowments of managerial talent, nx grows while n^*x^* shrinks. The reduction in aggregate output by Western firms enhances their incentive to offshore tasks to East as compared to performing them at smaller scale at home. Thus, I grows. The expansion of the set of tasks concentrated in the East spells an increase in demand for labor there, which bids up the relative wage w and causes more tasks with moderate offshoring costs to locate in West; that is, w and I^* increase with σ as well.

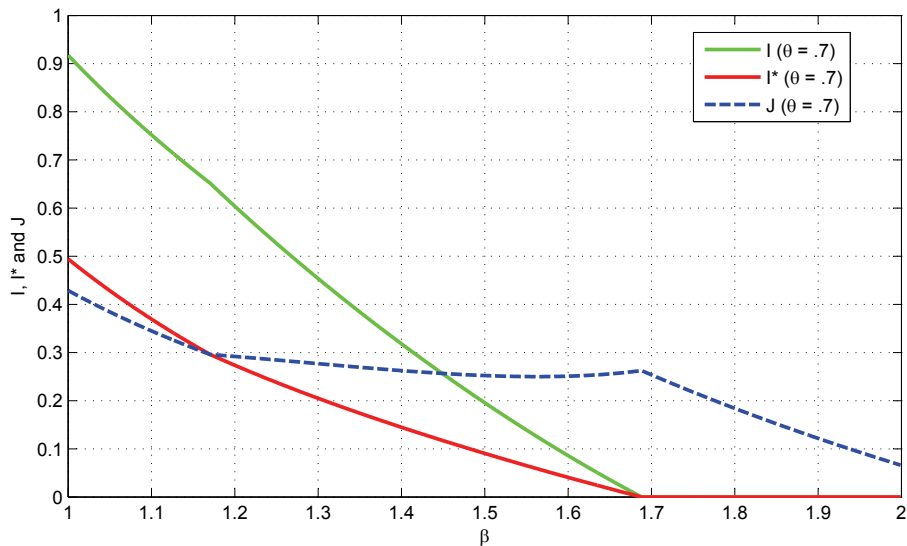
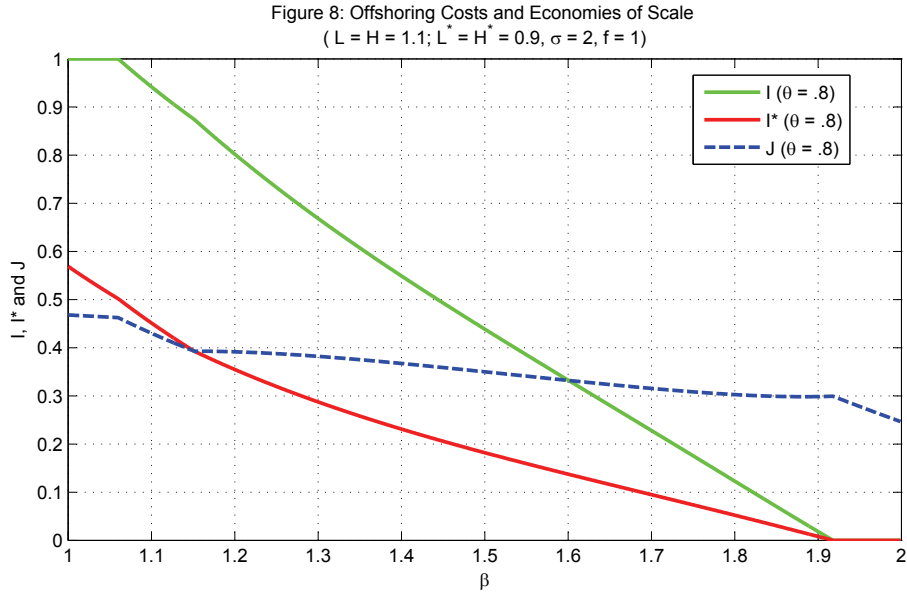
Now consider a division of resources such as $L = 1.45$, for which $I = 1$. Once the difference in country size and the elasticity of substitution are such that no tasks are performed in both countries, a further increase in product substitutability cannot expand the set \mathcal{E} at the expense of the set \mathcal{B} . Nonetheless, we find that a rise in σ increases the relative wage of the East due to the magnified scale advantage it derives from its lesser costs. In the bottom panel, w increases with σ , albeit to a lesser extent than for more equal country sizes. The rise in w diminishes Western firms' incentives to concentrate the marginal tasks in East, while the fall in the scale of aggregate output by Western firms has the opposite effect by reducing the productivity of dispersed production there. For the parameters that underlie Figure 7, the former effect (barely) dominates. Therefore, we see that I^* falls slightly as σ increases from 1.6 to 2.4.

4.3 Size of Offshoring Costs

Finally, we examine variations in offshoring costs that might result, for example, from improvements in communication technology. Figures 8 and 9 are drawn for a given division of the world's resources with $L = H = 1.1$ and $L^* = H^* = 0.9$. Both figures assume $\sigma = 2$ and $f = 1$ and both show features of the equilibrium for two different values of θ , namely $\theta = 0.7$ and $\theta = 0.8$. Finally, the figures display outcomes for offshoring costs ranging between $\beta = 1$ and $\beta = 2$.

The two panels of Figure 8 show I , I^* and J for the alternative values of the scale economy parameter θ . Consider first the uppermost panel, which depicts the case with greater scale economies. When offshoring is not very costly, $I = 1$, which means that all tasks are concentrated in one country or the other. The tasks with $i \leq J$ are performed in West, while those with $i > J$ are performed in East. In this case, aggregate production plus offshoring costs are minimized for every task. This is possible, because the strong scale economies make concentration desirable and the modest offshoring costs make deviations to dispersed production relatively unappealing. For tasks $i > J$, concentrated production in West is undermined by a deviant to East who sets prices sufficiently low to attract all producers as customers. The boundary at I^* plays no role in the equilibrium,

because the tasks just above and just below I^* are concentrated in East by dint of having indexes greater than J , so the fact that a deviation to East would be profitable were they instead to be concentrated in West is irrelevant.



As offshoring costs increase, the thresholds I and I^* fall. The greater offshoring costs enhance the profitability of a deviant who serves only local producers. For $\beta = 1.1$, it is no longer possible to concentrate the performance of all tasks. Rather, those with $i \leq J$ are concentrated in West, those with $J < i \leq I$ are concentrated in East, and those with $i > I$ are performed locally in both countries. For $\beta = 1.2$, the threshold J no longer plays a role in the allocation of tasks, as I^*

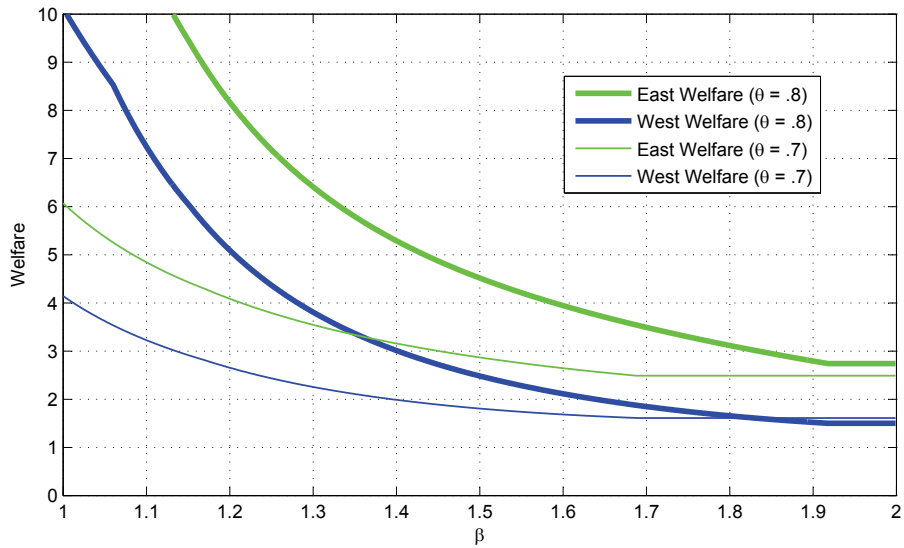
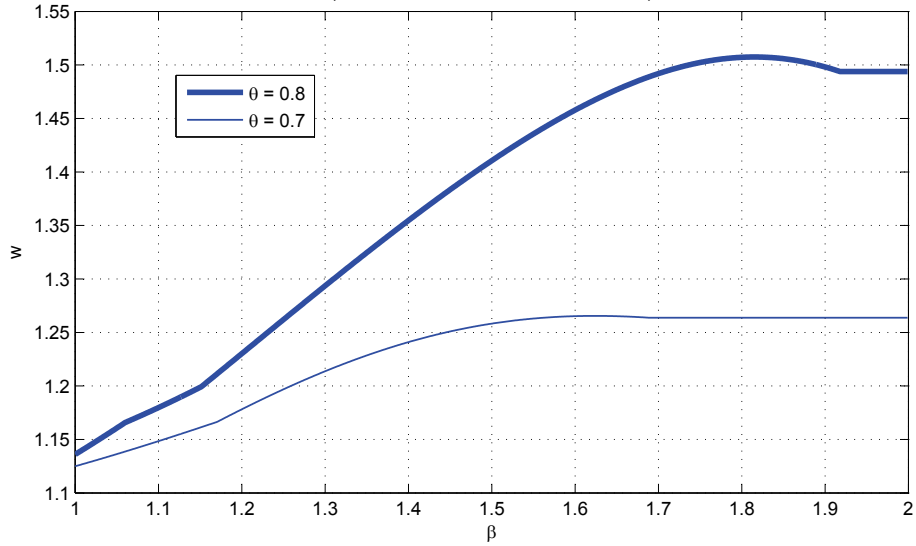
instead determines the boundary between tasks that are concentrated in West and in East. Still further increases in β reduce the sets of tasks concentrated in each country and expand the set that is dispersed. Finally, for β greater than 1.92, offshoring does not take place. As noted previously, the case of $\sigma = 2$ that is depicted here has I and I^* reaching zero for the same value of β , which means that each country's offshore production ceases when its hosting of foreign producers does as well.

Comparing the top and bottom panels of Figure 8, we see again that stronger scale economies imply greater concentration of tasks. The set \mathcal{W} , which comprises tasks with $0 \leq i \leq \min[I^*, J]$, is larger for every value of β in the top panel than in the bottom panel, as is the set \mathcal{E} , which comprises tasks with $\min[I^*, J] < i \leq I$. Otherwise, the bottom panel is qualitatively similar to the top.

As the offshoring costs increase, the relative wage of the East first increases, then decreases slightly, and ultimately becomes constant (see the top panel of Figure 9). The increase in the relative wage reflects the fact that the West suffers more from higher offshoring costs due to its smaller scale for dispersed tasks. This effect is more pronounced when scale economies are strong, which explains why the thick curve rises more steeply than the thinner curve. Once the offshoring costs reach a level such that few tasks are traded, another effect dominates. Workers in the East benefit relative to those in the West from the fact that their country specializes in tasks that are more costly to offshore. But this relative benefit disappears as β grows large. Finally, when offshoring costs are so high as to choke off task trade, further increases in these costs have no bearing on the countries' wages.

The bottom panel of Figure 9 tells a related story about aggregate welfare. We have found a monotonic relationship between offshoring costs and welfare in each country for a wide range of parameter values. The adverse effects of higher offshoring costs are especially pronounced when θ is large, because the potential gains from specialization are greater in such circumstances. The figure shows that the West fares better when scale economies are strong than when they are weak (compare the thick and thin dark curves) when β is low, but the opposite is true (albeit barely so) when β is large. When β is small, the West gains from trading a wide range of tasks, the more so the stronger are the increasing returns to scale. But when offshoring costs are high, little or no task trade takes place. Then a strengthening of scale economies reduces the relative cost of final goods in the larger East, which in turn induces consumers to substitute toward their goods. The scale of production in the small West can fall as θ increases, in which case its welfare may decline.

Figure 9: Offshoring Costs and Economies of Scale
 ($L = H = 1.1$; $L^* = H^* = 0.9$, $\sigma = 2$, $f = 1$)



5 Concluding Remarks

We have developed a theory of task trade between similar countries. When offshoring costs are not too high, firms concentrate certain tasks in particular locations in order to realize external economies of scale. The potential for outsourcing allows them to overcome some aspects of the coordination problem inherent in this. Our theory predicts the pattern of specialization by task for countries that differ only in size. We find that there always exists an equilibrium in which the larger country has higher wages and greater aggregate output of final goods. If offshoring costs are low

enough and the countries are not too different in size, there may exist another equilibrium in which the smaller country has the higher wages and greater aggregate output. In either case, the country with the higher wages and output performs the tasks—among those that are concentrated—that are more difficult and costly to offshore.

Our main empirical prediction links the pattern of specialization in tasks to relative wages. To test this prediction, we would need to identify the characteristics of tasks performed in different countries, which is by no means an easy thing to do. However, Autor et al. (2003) have shown that it is possible to distinguish the tasks performed in a country using data on the distribution of workers across occupations and information about the type of work performed by individuals in each narrowly-defined occupational category. They have measured the specialization of the U.S. economy across five task categories: routine and manual, routine and cognitive, non-routine and interactive, non-routine and analytic, and non-routine and manual. Since the 1980's, the United States has been specializing more in tasks that are non-routine and either interactive or analytic, and less in the other three categories of tasks.

Spitz-Oener (2006) has conducted a similar exercise using German data. She finds that the pattern of specialization across tasks has evolved similarly in Germany as in the United States, except that Germany is performing more tasks that are non-routine and manual over time, unlike the United States. The evidence supports the plausible conclusion that routine tasks are migrating to low-income countries like China, India and Mexico, with the high-income countries specializing increasingly in the set of non-routine tasks. But the evidence also suggests that Germany is specializing in a different set of non-routine tasks than the United States, namely, those that are more manual in nature. Given that Germany is smaller than the United States in terms of aggregate output and it has lower wages, our theory predicts that it should specialize in tasks that are relatively easier to offshore. Our prediction accords with the available evidence to the extent that (non-routine) manual tasks can more readily be organized and coordinated from a distance than interactive or analytic tasks. This ranking of relative offshoring costs seems plausible to us, but we could find no direct evidence to confirm it.

Ideally, empirical research on task trade would begin by classifying tasks according to the relative ease of offshoring. More data on offshoring are becoming available as awareness of this phenomenon grows, so it may soon be possible to measure the offshoring costs for different tasks. Once that is possible, it will also be possible to study the pattern of specialization by task. We hope that our theory can help guide such efforts.

6 Appendix

We first prove the claim we made in Section 2.1. We then proceed to the proofs of Lemmas 1 and 2.

Claim 1 *Either (i) $\pi_d(i) > 0$ or $\pi_d(i) < 0$ for all i , or (ii) $J > 0$ and $\pi_d(i) > 0$ for $i < J$ while $\pi_d(i) < 0$ for $i > J$.*

Proof. Without loss of generality assume $w > 1$. The aggregate cost of performing task i in the East minus the aggregate cost of performing it in the West is proportional (since it is divided by $A(nx + n^*x^*)$) to

$$\begin{aligned}\Lambda(i; nx, n^*x^*, w) &\equiv \pi_d(i; nx, n^*x^*, w) A(nx + n^*x^*) \\ &= wnx + \beta t(i) wn^*x^* - (n^*x^* + \beta t(i) nx) \\ &= (wnx - n^*x^*) - \beta t(i) (nx - wn^*x^*).\end{aligned}$$

First assume that $n^*x^* \geq nx$. Then $nx - wn^*x^* < 0$ which implies $\min_i \Lambda(i; nx, n^*x^*, w) = \Lambda(0; nx, n^*x^*, w)$ since $t'(i) > 0$ for all i . Then, since $\beta t(0) > 1$,

$$\begin{aligned}\Lambda(0; nx, n^*x^*, w) &> wnx - n^*x^* - nx + wn^*x^* \\ &= (w - 1)(nx + n^*x^*) > 0.\end{aligned}$$

So all tasks have higher aggregate cost in the East; i.e. $\pi_d(i) > 0$ for all i and $J = 1$.

Now suppose instead that $nx > n^*x^*$. Then $wnx - n^*x^* > nx - wn^*x^*$. Suppose first that $\beta t(0) > 1$ is close enough to one that $\Lambda(0; nx, n^*x^*, w) > 0$. Then tasks in the neighborhood of task 0 yield lower costs in the West. Since $t'(i) > 0$ for all i , either there exists $J > 0$ such that $\Lambda(J; nx, n^*x^*, w) = 0$, in which case tasks with $i > J$ have lower cost in the East ($\pi_d(i) < 0$) and tasks with $i < J$ have lower cost in West ($\pi_d(i) > 0$), or $(wnx - n^*x^*) > \beta t(1)(nx - wn^*x^*)$ in which case $\Lambda(i; nx, n^*x^*, w) > 0$ for all i and all tasks have lower cost in the West ($\pi_d(i) > 0$ and $J = 1$). If $\beta t(0)$ is such that $\Lambda(0; nx, n^*x^*, w) < 0$, then since $t'(i) > 0$ for all i , all tasks have lower costs in the East, namely, $\pi_d(i) < 0$ and $J = 0$. ■

Lemma 1 *If $w > 1$, $J < I$ implies $I > I^*$.*

Proof. The proof of Claim 1 in the Appendix guarantees that if $w > 1$ then $n^*x^* > nx$ implies $J = 1$. So we can limit our attention to circumstances with $nx > n^*x^*$. Note that

$$\beta t(I^*) = \frac{wA(nx + n^*x^*)}{A(nx)}$$

and

$$\beta t(I) = \frac{A(nx + n^*x^*)}{wA(n^*x^*)}.$$

To establish a contradiction, we suppose that $J < I$ and $I^* > I$. Then

$$\frac{wA(nx + n^*x^*)}{A(nx)} > \frac{A(nx + n^*x^*)}{wA(n^*x^*)}$$

and

$$w^2 > \frac{A(nx)}{A(n^*x^*)}. \quad (13)$$

From the definition of J , we know that

$$\beta t(J) - \beta t(I) = \frac{wnx - n^*x^*}{nx - wn^*x^*} - \frac{A(nx + n^*x^*)}{wA(n^*x^*)}. \quad (14)$$

Since the denominators are both positive for $J \in (0, 1)$, the left-hand side has the same sign as

$$\begin{aligned} & \Delta(n^*x^*, nx, w) \\ \equiv & w^2A(n^*x^*)nx - wA(n^*x^*)n^*x^* - A(nx + n^*x^*)nx + wA(nx + n^*x^*)n^*x^* \end{aligned}$$

But then (13) implies that

$$\begin{aligned} & \Delta(n^*x^*, nx, w) \\ > & A(nx)nx - wA(n^*x^*)n^*x^* - A(nx + n^*x^*)nx + wA(nx + n^*x^*)n^*x^* \\ = & wn^*x^*[A(nx + n^*x^*) - A(n^*x^*)] + nx[A(nx) - A(nx + n^*x^*)] \\ > & n^*x^*[A(nx + n^*x^*) - A(n^*x^*)] + nx[A(nx) - A(nx + n^*x^*)]. \end{aligned}$$

Define the last term on the right-hand side as

$$\Omega(n^*x^*, nx) \equiv (n^*x^* - nx)A(nx + n^*x^*) - n^*x^*A(n^*x^*) + nxA(nx),$$

and note that $\Omega(\cdot)$ is continuously differentiable in both arguments and

$$\Omega(nx, nx) = 0.$$

Calculate the partial derivative of $\Omega(n^*x^*, nx)$ with respect to the second argument,

$$\Omega_2(n^*x^*, nx) = A(nx) + nxA'(nx) - (nx - n^*x^*)A'(n^*x^* + nx) - A(n^*x^* + nx).$$

Then

$$\Omega_2(0, nx) = 0$$

and

$$\Omega_2(nx, nx) = A(nx) + nxA'(nx) - A(2nx) \geq 0,$$

where the inequality follows from the concavity of $A(\cdot)$. Note also that

$$\Omega_{12}(n^*x^*, nx) = -(nx - n^*x^*)A''(n^*x^* + nx) \geq 0,$$

by the concavity of $A(\cdot)$. Then, since $\Omega_2(\cdot)$ is continuous, $\Omega_2(n^*x^*, nx) \geq 0$ for all $n^*x^* \geq 0$ and $nx \geq n^*x^*$. Since $\Omega(nx, nx) = 0$ and $\Omega_2(n^*x^*, nx) \geq 0$ for all $nx \geq n^*x^*$, it follows by continuity that $\Omega(n^*x^*, nx) \geq 0$ for all $nx \geq n^*x^*$. Hence, if $w > 1$, $I^* > I$, and $nx > n^*x^*$, we obtain that $\Delta(n^*x^*, nx, w) > 0$, which implies by (14) that $J > I$. This establishes our contradiction. ■

Lemma 2 $w > 1$ if and only if $nx > n^*x^*$.

Proof. We consider three mutually exhaustive cases: (i) $I \geq I^*$, (ii) $I < I^*$ and $L > L^*$, and (iii) $I < I^*$ and $L \leq L^*$.

(i) From the definitions of I and I^* ,

$$\frac{\beta t(I)w}{A(nx + n^*x^*)} \leq \frac{1}{A(n^*x^*)}$$

and

$$\frac{\beta t(I^*)}{A(nx + n^*x^*)} \geq \frac{w}{A(nx)}.$$

The first inequality can strict when $I = 1$, the second when $I^* = 0$. Therefore

$$\frac{A(nx + n^*x^*)}{wA(n^*x^*)} \geq \beta t(I) \geq \beta t(I^*) \geq \frac{wA(nx + n^*x^*)}{A(nx)},$$

which implies that

$$\frac{A(nx)}{A(n^*x^*)} \geq w^2 > 1.$$

So $nx > n^*x^*$.

(ii) To establish a contradiction, suppose that $nx \leq n^*x^*$. From Figure 5, $I < I^*$ implies $\mathcal{E} = \emptyset$.

Then

$$L = \frac{M(\mathcal{B})nx}{A(nx)} > L^* > \frac{M(\mathcal{B})n^*x^*}{A(n^*x^*)}$$

which implies

$$\frac{A(nx)}{nx} < \frac{A(n^*x^*)}{n^*x^*}.$$

But $A(\cdot)$ concave, $A(0) \geq 0$, and $nx \leq n^*x^*$, imply that

$$\frac{A(nx)}{nx} \geq \frac{A(n^*x^*)}{n^*x^*}.$$

This contradicts the supposition that $nx < n^*x^*$.

(iii) To establish a contradiction, suppose that $nx \leq n^*x^*$. Labor-market clearing implies

$$L = (1 - I^*) \frac{nx}{A(nx)}$$

and

$$L^* > (1 - I^*) \frac{n^*x^*}{A(n^*x^*)} + I^* \frac{nx + n^*x^*}{A(nx + n^*x^*)},$$

since $T(I^*) > I^*$ for all I^* .

From manager-market clearing, and $H = L$ and $H^* = L^*$, this implies that

$$\frac{n^*}{n} = \frac{L^*}{L} > \frac{(1 - I^*) \frac{n^*x^*}{A(n^*x^*)} + I^* \frac{nx + n^*x^*}{A(nx + n^*x^*)}}{(1 - I^*) \frac{nx}{A(nx)}}$$

Multiplying both sides by $nx/(n^*x^*)$, we obtain

$$\frac{x}{x^*} > \frac{\frac{1-I^*}{A(n^*x^*)} + I^* \left(\frac{nx+n^*x^*}{n^*x^*} \right) \frac{1}{A(nx+n^*x^*)}}{\frac{1-I^*}{A(nx)}}.$$

Note that $nx \leq n^*x^*$ and $w > 1$ imply that

$$\frac{c}{c^*} = \frac{\frac{w(1-I^*)}{A(nx)} + \frac{\beta T(I^*)}{A(nx+n^*x^*)}}{\frac{1-I^*}{A(n^*x^*)} + \frac{I^*}{A(nx+n^*x^*)}} \geq 1.$$

CES preferences and goods-market clearing yield

$$\frac{x^*}{x} = \left(\frac{c}{c^*} \right)^\sigma,$$

and since $\sigma > 1$, this implies

$$\frac{x^*}{x} \geq \frac{c}{c^*}.$$

Given that $T(I^*) > I^*$ and $w > 1$

$$\frac{x^*}{x} \geq \frac{c}{c^*} > \frac{\frac{1-I^*}{A(nx)} + \frac{I^*}{A(nx+n^*x^*)}}{\frac{1-I^*}{A(n^*x^*)} + \frac{I^*}{A(nx+n^*x^*)}}$$

or

$$\frac{x}{x^*} < \frac{\frac{1-I^*}{A(n^*x^*)} + \frac{I^*}{A(nx+n^*x^*)}}{\frac{1-I^*}{A(nx)} + \frac{I^*}{A(nx+n^*x^*)}}.$$

Therefore, in order for an equilibrium to exhibit $nx < n^*x^*$ it has to be the case that

$$\frac{\frac{1-I^*}{A(n^*x^*)} + \frac{I^*}{A(nx+n^*x^*)}}{\frac{1-I^*}{A(nx)} + \frac{I^*}{A(nx+n^*x^*)}} > \frac{x}{x^*} > \frac{\frac{1-I^*}{A(n^*x^*)} + I^* \left(\frac{nx+n^*x^*}{n^*x^*} \right) \frac{1}{A(nx+n^*x^*)}}{\frac{1-I^*}{A(nx)}}.$$

But note that $I^*/A(nx + n^*x^*) > 0$ and $(nx + n^*x^*)/n^*x^* > 1$ so

$$\frac{\frac{1-I^*}{A(n^*x^*)} + \frac{I^*}{A(nx+n^*x^*)}}{\frac{1-I^*}{A(nx)} + \frac{I^*}{A(nx+n^*x^*)}} < \frac{\frac{1-I^*}{A(n^*x^*)} + I^* \left(\frac{nx+n^*x^*}{n^*x^*} \right) \frac{1}{A(nx+n^*x^*)}}{\frac{1-I^*}{A(nx)}},$$

which contradicts the previous string of inequalities. ■

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